Physics for School Certificate

(HEAT, LIGHT AND SOUND).

A REVISION COURSE.

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Physics for School Certificate

(HEAT, LIGHT AND SOUND).

A REVISION COURSE

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No.....
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THIRD EDITION, 1929.

EXETER:

A. WHEATON & CO., Ltd., Educational Publishers.

1928.

MULLER PATEN BROS.
BOOKSELLERS, STATIONERS AND AGENTS
BRIGADE ROAD BANGALORE

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PREFACE.

THE kind reception accorded to my *Chemistry for School Certificate : A Revision Course*, showed that there existed a real need for a book of that type, and encourages me to hope that a similar book on Physics may also be welcomed.

In some subjects (e.g., Mathematics and French) the pupils may be said to be continuously revising all the work that they have previously done. In Physics and Chemistry this is by no means the case, and the difficulty of bringing up to examination standard the whole of the work done in a four years' course is a very serious one. In Physics indeed this full revision is seldom attempted, and the pupil usually elects to be examined in only a part of the work he has actually studied. Even so, the task of revision is still a formidable one, and the teacher has often to curtail a really progressive course in order to devote sufficient time to it.

It is hoped that the present volume will greatly simplify the task. By giving additional or alternative notes where necessary, the teacher will be able to make the book fit more exactly the needs of his own course. Assuming that this is done, and that the matter contained in the book is supplemented by proper class lessons and directions for practical work, no further *formal* text-book should be necessary, though a pupil should certainly read books from his School Science Library dealing with the applications of Physics to daily life. However, to meet the needs of teachers who would prefer to retain one of the larger text-books and use the *Revision Course* as a supplementary book, the price has been kept as low as possible.

During the preparation of the book I addressed a circular letter to considerably over 1,000 science teachers

asking for suggestions calculated to make it more useful. Some of the replies received were most helpful, and I take this opportunity of expressing my very sincere thanks. I must also thank the various authorities concerned for their kind permission to include questions set in recent years at the School Certificate Examinations arranged by the Universities of Cambridge, London and Oxford, by the Central Welsh Board, the Oxford and Cambridge Schools Examination Board, the Matriculation Joint Board, and the Scottish Education Department (for the last mentioned, the permission of H.M. Stationery Office has been obtained).

In a book which is intended to serve as a revision course, it is obvious that little or none of the matter can be original, and the question of making a due acknowledgment of one's sources becomes difficult indeed. Speaking generally, however, I may say that the experiments described are those which I used for many years in my own classes in Physics, and which I think may fairly be regarded as "common stock."

My colleagues, Mr. Crane, Mr. Giles and Mr. Hayward, helped me by working out the numerical examples. Mr. Crane also read through the whole of the proofs and made some useful suggestions. I greatly appreciate their kind help. Finally, I must express my gratitude to W. Clarke, R. Collins, L. Hunt, C. Osman and M. Pitts (of the Lower Fifth) who made detailed corrections of the proof sheets.

W. LITTLER.

Hele's School, Exeter,
April, 1928.

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Physics for School Certificate.

Chapter I.

THERMOMETERS. EXPANSION OF SOLIDS.

1. Marking the Fixed Points on a Thermometer.

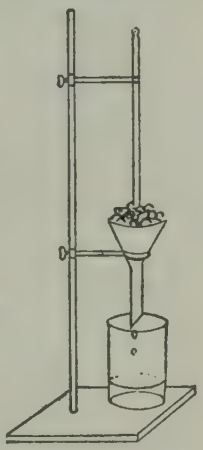

Fig. 1. Marking Freezing Point.

(a) *Lower Fixed Point.* The thermometer is surrounded by clean crushed ice in a funnel (fig. 1), and is raised or lowered until the thread of mercury is just visible above the ice. When the mercury column has become quite steady, the point is marked on the stem by means of a file scratch.

(b) *Higher Fixed Point.* The temperature at which water boils depends on several factors:—

(i.) The degree of purity of the water.

(ii.) The external pressure.

(iii.) To a slight extent on the nature of the containing vessel.

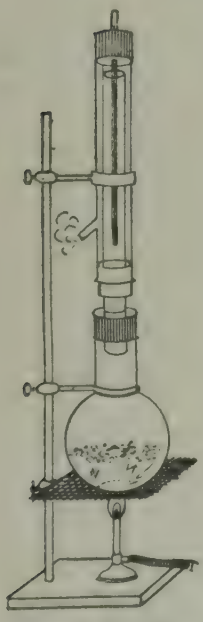

Fig. 2. Marking Boiling Point.

The temperature of the *steam* from boiling water, however, depends only on the external pressure. Hence in marking the higher fixed point, the thermometer is surrounded with steam. The apparatus used is called a *hypsometer*, one form being shown in fig. 2.

Note that the thermometer is surrounded by a double jacket of steam; thus the inner jacket is protected from the cooling action of the air.

The boiling point is corrected in accordance with the fact that in the neighbourhood of the ordinary boiling point an increase of 28 mm. in the pressure raises the boiling point 1° C. Thus, suppose the pressure at the time of the

experiment were 769 mm. (i.e. 9 mm. above normal), this would produce a rise in the boiling point of $\frac{9}{28}$ or 0.3° C., and the observed temperature would be taken not as 100° C., but as 100.3° C.

Since alcohol boils at 78° C., it would evidently be impossible to treat an alcohol thermometer in the way just described. An alcohol thermometer may, however, be graduated by comparison with a mercury thermometer.

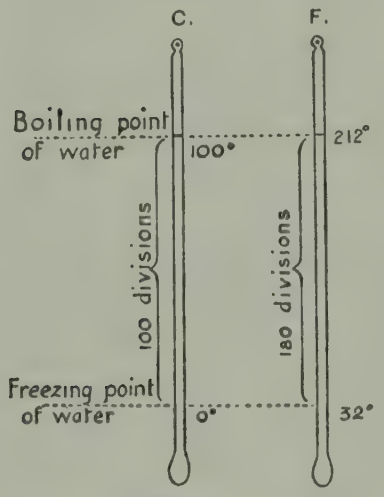

Fig. 3.
Centigrade and Fahrenheit Thermometers compared.

2. Fahrenheit and Centigrade Thermometers. The fixed points on these thermometers are numbered as shown in fig. 3. It will be seen that

100 Centigrade divisions
= 180 Fahrenheit divisions.

∴ 1 Centigrade division
= $\frac{9}{5}$ Fahrenheit division.

We are now in a position to bring Fahrenheit temperatures to Centigrade, and vice versa.

Example 1. What Fahrenheit temperature corresponds to 20° Centigrade ?

20 C. divisions above freezing point.
= $\frac{9}{5}$ × 20 F. divisions above freezing point
= 36 F. divisions above freezing point,
But " freezing point " is itself 32° F.
∴ 36 F. divisions above this will be (36 + 32) F. = **68° F.**

Example 2. What Centigrade temperature corresponds to 77° Fahrenheit ?

We must first find how much 77° F. is above freezing point.
The latter is 32° F.
∴ 77° F. is (77—32) or 45 Fahrenheit divisions above freezing point.
But 45 Fahrenheit divisions = $\frac{5}{9}$ × 45 or 25 Centigrade divisions.
∴ required temperature is **25° C.**

Examples such as 1 and 2 lead up to the following

Rules. To bring degrees Centigrade to Fahrenheit, multiply by $\frac{9}{5}$ and add 32. To bring degrees Fahrenheit to Centigrade, first subtract 32 and then multiply by $\frac{5}{9}$.

Example 3. At what temperature would a Centigrade and a Fahrenheit thermometer give the same reading?

Supposed required temperature is x degrees F.
Now, by the above rule, $x°$ F. $= (x—32) \times \frac{5}{9}$ degrees C.
But $x°$ F. $= x°$ C. (for the thermometers are giving the same reading).
$\therefore (x—32) \times \frac{5}{9} = x$.
Solving this equation, we find that **$x = —40°$.**

3. Mercury and Alcohol as Thermometric Liquids. As a thermometric liquid, mercury possesses many advantages :—

(i.) High boiling point (357° C.) and fairly low freezing point (—39° C.)

(ii.) It expands very regularly.

(Hence the interval between freezing point and boiling point on a Centigrade thermometer may be divided into 100 *equal* parts.)

(iii.) Being opaque, it is easily seen.

(iv.) It does not wet the tube.

(v.) It is a good conductor, and therefore the whole of the liquid in the bulb quickly attains the required temperature.

" Low specific heat " is usually mentioned in the list of advantages, but this is practically counterbalanced by the higher density of mercury (see Example 18, §61).

Its chief *disadvantage* is its very low coefficient of expansion, which makes it necessary to use a tube of very fine bore.

[Q.—Can you see any advantage that will follow from this ?—Remember that because the bore is fine, an unusually small amount of the liquid will be present in the exposed stem.]

Alcohol suffers from the rather serious disadvantage of a low boiling point (78° C. or 172.4° F.), but its very low freezing point (—130° C.) has caused it to be much used by Polar explorers.

Its rate of expansion *increases* with rise of temperature.

[Q.—How could this difficulty be surmounted ?]

It is colourless and transparent like water, and would therefore be very difficult to see.

[Q.—Remedy ?]

Further, it wets the tube in which it is contained.

[Q.—How does this affect the readings ?]

and it is a poor conductor.

For these reasons, alcohol thermometers, though largely employed for domestic purposes, are seldom used where great accuracy is required.

4. Thermometers for Special Purposes.

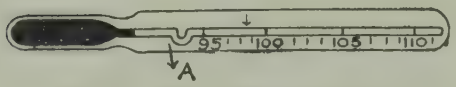

Fig. 4. Clinical Thermometer.

(i.) *Clinical Thermometer.* This is used by the doctor in taking the temperature of the human body, which in health never varies much from 98.4° F. Its special feature is a *constriction in the bore of the tube* (at A, fig. 4).

Thus when the thermometer has taken the required temperature and is being removed for inspection, the mercury thread breaks off at A. Hence the upper portion remains stranded and gives the correct reading. After it has been read it is shaken, and this causes the separated parts of the mercury column to re-unite.

Another feature is its *small range* (usually 95° F. to 110° F.). This enables us to use the whole length of the stem for a range of some 15 degrees only, and hence the divisions can be so large that they can easily be subdivided into fifths.

[Q.—What difference would you expect to find between the bore of the tube of a clinical thermometer and that of an ordinary thermometer ? If there were no difference in the bore, what modifications would have to be made in the bulb ?]

(ii.) *Maximum and Minimum Thermometer.* This is designed to show the maximum or minimum temperature recorded during a given interval. Fig. 5 shows a form

known as *Six's*, in which the two thermometers are combined. B is the bulb, and the thermometric liquid, extending from A to E, consists of alcohol. C and D are steel floats clinging by means of very weak springs to the walls of the tube (fig. 5a).

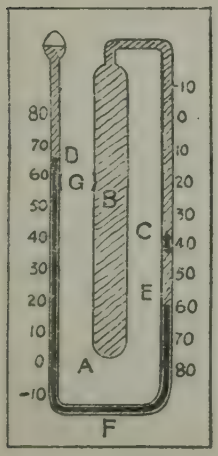

Fig. 5.
Six's Maximum and Minimum Thermometer.

Fig. 5a.

With rising temperature, the alcohol flows past C and pushes the mercury column EFG in front of it, and the end G in turn pushes the spring D. A little thought will show that the maximum temperature during the interval is recorded by the lower end of D, while the minimum temperature is recorded by the lower end of C. To re-set the instrument, a small magnet is used.

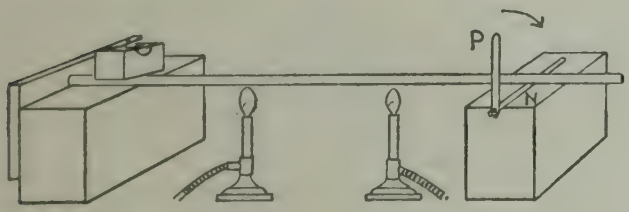

Fig. 6. Expansion of Iron on Heating.

5. Expansion of Solids. In fig. 6 is indicated a common way of showing that iron expands on heating. A very slight expansion of the rod of the retort-stand causes the needle N to roll slightly, and this causes a considerable angular movement of the pointer P.

If a flask filled with water is arranged as in fig. 7 and then suddenly plunged into hot water, the glass of the flask becomes heated for a moment while the

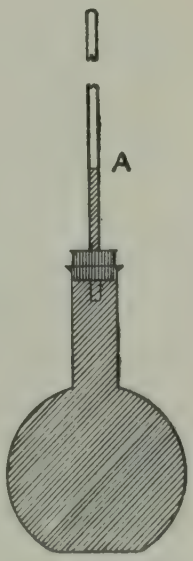

Fig. 7.
Expansion of Glass and of Water.

contained water is still cold, and the water level at A *falls*. After a second or two, when the water in the flask has had a chance to become heated, the water level *rises* beyond its previous mark.

[Q.—What conclusion may we draw as regards the comparative expansion of glass and water ?]

6. Coefficient of Linear Expansion. *Definition*—**Suppose a solid substance of length x has its temperature raised by 1 degree Centigrade, and that as a result it becomes longer by an amount y. Then the ratio y/x is known as the *coefficient of linear expansion* of that substance.***

7. Expansion of Unit Length.

Since x cms. (when heated through 1° C.) expand by y cms.,

$\therefore$ 1 cm. (when heated through 1° C.) expands by $\frac{y}{x}$ cms.

But $\frac{y}{x}$ is the coefficient of linear expansion.

Hence *the coefficient of linear expansion is numerically equal to the expansion of* 1 *cm. length of the substance for a rise in temperature of* 1 *degree Centigrade.*

Thus when we read that the coefficient of linear expansion of copper is .000017, we understand

(a) That any length of copper, when heated through 1° C., will expand by .000017 of its original length,

or (b) That a copper rod (or wire) 1 cm. long, when heated through 1° C., will expand by .000017 cm.

[Q.—How would this number be affected, (i.) if all measurements were made in inches instead of in cms. ; (ii.) if the Fahrenheit scale had been used instead of the Centigrade ?]

8. Coefficient of Cubical Expansion. *Definition*—**Suppose a substance of volume x has its temperature raised by 1 degree Centigrade, and that as a result its volume becomes greater by an amount y. Then the ratio y/x is known as the *coefficient of cubical expansion of that substance.***

* *The student is advised to learn by heart all statements in heavy type.*

Coefficient of superficial (or surface) expansion is similarly defined.

9. Relation between Cubical and Linear Coefficients. Consider a cube of a substance whose coefficient of linear expansion is α. Let each side of the cube measure exactly 1 cm. Clearly the volume of the cube is 1 cubic centimetre.

Now let the cube be heated through 1° C. Each edge now measures $(1 + \alpha)$ cms.

$\therefore$ new volume $= (1 + \alpha)^3 = 1 + 3\alpha + 3\alpha^2 + \alpha^3$ c.c.

Now α is a very small quantity (e.g., .000017 in the case of copper).

$\therefore$ $3\alpha^2$ and α^3 may be neglected in comparison with α (e.g., in the case of copper, $3\alpha^2 = 3 \times .000017^2 = .00000000087$ approx., and α^3 is smaller still).

$\therefore$ new volume $= 1 + 3\alpha$ c.c. approx.

$\therefore$ *increase* in volume $= 3\alpha$ c.c.

But the increase in volume of 1 c.c. for a rise in temperature of 1° C. is the coefficient of cubical expansion (cf. §6).

$\therefore$ coefficient of cubical expansion (3α) is numerically equal to three times the coefficient of linear expansion.

It is left as an exercise to the student to show that the coefficient of surface expansion is numerically equal to twice the coefficient of linear expansion.

10. Some Convenient Formulæ. Suppose the length of a bar of a certain substance is l_0 at 0° C. and l_t at t° C. We require to find a connection between l_t and l_0.

Let α = coefficient of linear expansion.

The expansion of each centimetre of the bar for a rise of 1° C. is α cms.

$\therefore$ for a rise of t° C. the expansion is αt cms.

$\therefore$ 1 cm. at 0° C. becomes $1 + \alpha t$ cms. at t° C.

$\therefore$ l_0 cms. at 0° C. become $l_0 (1 + \alpha t)$ cms. at t°C.

i.e., $l_t = l_0 (1 + \alpha t)$.

Similarly if V_0 = volume at 0° C., V_t = volume at t° C., and c = coefficient of cubical expansion, we can show that $V_t = V_0 (1 + ct)$.

11. Example 4. A steel ball has a volume of 10 c.c. at 0° C. Find its volume at 100° C. ($\alpha = .000012$).

$\alpha = .000012$.
$\therefore$ $c = .000012 \times 3$ (§9) $= .000036$.
1 c.c. for a rise of 1° C. expands .000036 c.c.
$\therefore$ 10 c.c. for a rise of 100° C. expands $.000036 \times 10 \times 100 = .036$ c.c.
$\therefore$ required volume = **10.036 c.c.**
Or applying the formula $V_t = V_o\ (1 + ct)$,
We have $V_o = 10$, $c = .000036$, $t = 100$.
$\therefore$ $V_{100} = 10(1 + .000036 \times 100)$.
$= 10 \times 1.0036 =$ **10.036 c.c.**

Example 5. A brass yard measure is exactly right at 15° C. Find its length at 50° C. ($\alpha = .000019$).

We do not know l_o, but applying the formula, we have $l_{50} = l_o\ (1 + \alpha t) = l_o\ (1 + \alpha \times 50)$
and $l_{15} = l_o\ (1 + \alpha \times 15)$.

$$\therefore \quad \frac{l_{50}}{l_{15}} = \frac{l_o(1 + 50\alpha)}{l_o(1 + 15\alpha)} = \frac{1 + 50\alpha}{1 + 15\alpha}$$

Substituting $l_{15} = 1$ (yard) and $\alpha = .000019$, we have

$$\frac{l_{50}}{1} = \frac{1 + 50 \times .000019}{1 + 15 \times .000019} = \frac{1.00095}{1.000285}$$

$\therefore$ $l_{50} =$ **1.00066 yards.**

Example 6. The length of a glass tube at 25° C. is 35.82 ins. according to a brass scale which is correct at 0° C. What is the true length of the tube, (a) at 25° C., (b) at 0° C. ?
[Coefficient of linear expansion of brass = 0.000019, of glass 0.000009].—*Lond.* 1921.

The part of the brass scale which lies against the glass tube measured 35.82 ins. at 0° C.
Applying the formula, $l_t = l_o\ (1 + \alpha t)$,
We have $\alpha = 0.000019$, $t = 25$, $l_o = 35.82$, l_t or $l_{25} = ?$
$\therefore$ $l_{25} = 35.82\ (1 + 0.000019 \times 25)$.
$= 35.82 \times 1.000475$.
$= 35.837$ inches.
Thus the part of the brass scale, and therefore the glass tube lying against it, measured **35.84 inches** at 25° C.
For the glass tube we now have $\alpha = 0.000009$, $t = 25$, l_t or $l_{25} = 35.837$, $l_o = ?$
Applying the formula $l_t = l_o\ (1 + \alpha t)$,
We have $35.837 = l_o\ (1 + 0.000009 \times 25)$,
On working out, we obtain $l_o =$ **35.829 inches.**

Example 7. If the specific gravity of copper at 0° C. is 8.95, find its specific gravity at 100° C. ($\alpha = .000017$).

1 c.c. for a rise of 1° C. expands $.000017 \times 3 = .000051$ c.c.

$\therefore$ 1 c.c. for a rise of 100° C. expands $.000051 \times 100 = .0051$ c.c.

i.e., the volume *increases* in the ratio $\frac{1.0051}{1}$.

$\therefore$ specific gravity *decreases* in the ratio $\frac{1}{1.0051}$

$\therefore$ required specific gravity is $8.95 \times \frac{1}{1.0051} = \mathbf{8.905}$

12. Measurement of α. A number of methods are in use for the measurement of α. One of them is indicated in fig. 8. AB is a tube of the substance under examination. It may be heated by sending in a current of

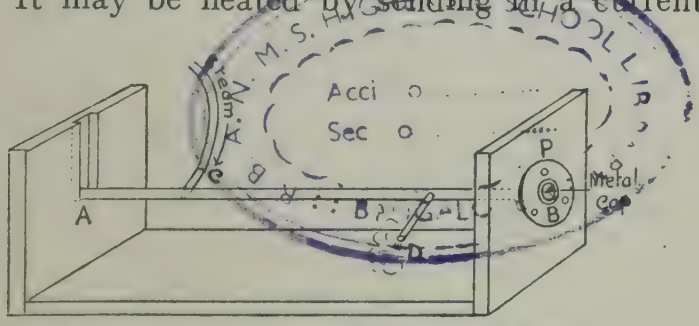

Fig. 8. Coefficient of Linear Expansion of a Metal.

steam which escapes at D. The tube rests in a frame, being rigidly fixed at the end A, but free to expand at B. This expansion takes place through a hole in the frame, which is surrounded by a flat metal plate, P, in which three small holes are drilled to take the three feet of a spherometer. The tip of the spherometer screw rests on a small metal cap which closes the end of the tube.

The steps in the experiment are as follows. The spherometer is placed in position and its reading is taken ; also the temperature of the room is noted and the length of the rod.

Steam is now passed through the tube for some time. The spherometer is again placed in position and a new reading is obtained. (To make sure that expansion is complete, the steam should be passed for an extra two

or three minutes, and the spherometer again applied. There should be no further change in the reading.) The result is then worked out as follows :—

Suppose length of tube = 120 cm.
Original temperature = 16° C.
Final temperature = 100° C.
First spherometer reading = 3.47 mm.
Second spherometer reading = 5.38 mm.
∴ 120 cms. when heated through 84° (100—16) expand .191 cm.

$$\therefore \text{1 cm. for 1° expands} = \frac{.191}{120 \times 84}$$

$$= .0000189 \text{ cm.}$$

i.e., coefficient of linear expansion = .0000189.

13. Unequal Expansion. If the above experiment is tried with a number of substances, different values of α are obtained. Thus for brass $\alpha = .000019$, while for iron it is .000012 and for glass .000009.

The fact that different substances have different coefficients of expansion may also be illustrated by the "compound bar" experiment.

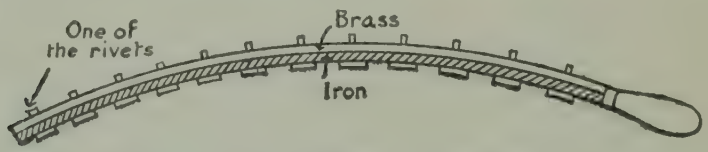

Fig. 9. Unequal Expansion of Brass and Iron.

Strips of iron and brass are firmly riveted together. On heating, the compound bar bends as illustrated in the figure, the brass being on the outside of the curve because it has the greater coefficient of expansion.

14. The Compensated Balance Wheel. The rate of a watch is controlled by a balance wheel, and without going into details, we may say that if the weight of the rim is thrown further out from the centre, the rate of oscillation is reduced, and vice versa. It is obvious then that in warm weather a watch that was previously keeping correct time will lose.

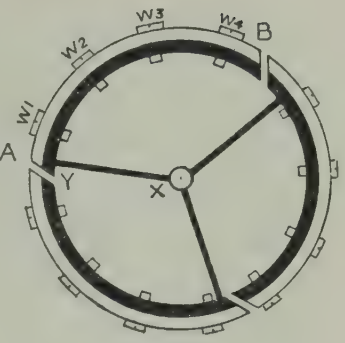

Fig. 10.
Compensated
Balance-wheel.

In a good watch this is prevented by constructing the rim in three segments, each consisting of brass and steel as shown in fig. 10. Consider the segment AB. With increase of temperature the spoke XY becomes longer, thus throwing the weight *further out* from the centre. But with increase of temperature, the segment acting as a compound bar becomes more curved, and the free end B is brought nearer to the centre. Thus the weight of the rim is brought *nearer to* the centre. By a proper adjustment of the screw weights W_1, W_2 . . . . these opposing tendencies can be made to neutralise one another, and the watch will continue to keep good time in spite of changed temperature.

In some cases the unequal expansion of different substances gives rise to inconvenience. Thus a metal wire cannot as a rule be fused into glass, because on cooling the metal contracts more than the glass and the latter cracks. It so happens, however, that platinum and glass have about the same coefficient, and so platinum is used in such cases.

15. Expansion a Convenience. The expansion of solid substances on heating is taken advantage of in the following cases among many others:—

(i.) *Loosening a glass stopper* which has stuck. This may be done by cautiously warming the neck.

(ii.) *The fire-alarm.* A powerful electric bell connected with the office, factory, bank, etc., forms part of an incomplete electric circuit which also includes a particular strip of metal. The latter is so arranged that when it becomes hot through the outbreak of fire, its expansion causes the gap in the circuit to be closed up and so sets the bell ringing.

(iii.) "*Shrinking on.*" The rim of a cart wheel, for instance, may be fitted very tightly on the wooden part by putting it on while it is hot.

16. Expansion an Inconvenience.

(i.) Cracking of glass vessel when suddenly heated (pouring in boiling water, etc.)

[Q.—Why do *thin* vessels not crack when so treated ? Why does a thick vessel not crack if *gradually* heated ?]

(ii.) Long iron bridges (e.g., the Forth Bridge) are perceptibly longer in summer than in winter, and the engineer has to make special arrangements to allow for this.

(iii.) The balance wheel of a watch has already been discussed.

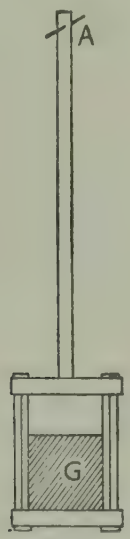

Fig. 11.
Compensated Pendulum.

(iv.) In warm weather the pendulum of a clock is a little longer, and therefore swings more slowly. One method of getting over this difficulty is illustrated in fig. 11. The length of the pendulum may be taken as the distance AG, i.e., from the point of suspension to the centre of gravity of the bob. Now if the bob is made to consist of a cylinder of mercury, of which G is the centre of gravity, it is evident that the expansion of the bar of the pendulum will cause G to *fall*, while the expansion of the mercury itself will cause G to *rise*. By adjusting the amount of mercury, these effects may be made to neutralise one another.

[Q.—What would you suggest doing, if a clock fitted with the pendulum just described, were found to *gain* in warm weather ?]

Clocks are now often made with their pendulums of *invar*, an alloy of nickel with a coefficient of expansion which is practically zero. Thus compensation is unnecessary.

QUESTIONS ON CHAPTER I.

Note.—The sources of the questions on this and all succeeding chapters are indicated as follows (School Certificate Examination in each case) :—

Camb. = Cambridge Local.
Lond. = London (General School).
Oxf. = Oxford Local.
C.W.B. = Central Welsh Board.
O.C. = Oxford and Cambridge Schools Examination Board.
M.L.L.S.B. = Matriculation Joint Board (Universities of Manchester, Liverpool, Leeds, Sheffield and Birmingham.)
Scot.L.C. = Scottish Leaving Certificate.

1. Turn the following into degrees Fahrenheit : 357° C. ; —39° C. ; 15° C. ; —273° C. ; 78° C.

2. Turn the following into degrees Centigrade : 60° F. ; —40° F. ; 238° F. ; 0° F. ; 98.4° F.

3. On a certain day the thermometer showed a rise of 27° F. between sunrise and noon. How many degrees would a Centigrade thermometer have risen ?

4. When a piece of ice was introduced into some water in a calorimeter, the temperature of the water fell from 26° C. to 6° C. *Write down* what this fall amounts to in degrees Fahrenheit.

5. The coefficient of linear expansion of iron is .000012 when all length measurements are made in centimetres, and when a Centigrade thermometer is understood to be employed. What value should we obtain for the coefficient if, (i.) all measurements were made in inches (Centigrade thermometer as before) ; (ii.) Fahrenheit degrees were substituted for Centigrade degrees (length measurements in cms. as before) ?

6. How much would 200 yards of copper wire expand if the temperature were raised 30° C. ? ($\alpha = .000017$.)

7. Find the volume at 80° C. of a zinc cube whose edge measures 10 cm. at 20° C. ($\alpha = .0000298$.)

8. The specific gravity of a specimen of steel is 7.82 at 0° C. Find its specific gravity at 80° C. ($\alpha = .000012$.)

9. Describe in detail the method you would use to find the position of the 100° C. mark on an ungraduated thermometer, explaining the reasons for any precautions against error. Convert 104° F. and 23° F. to the Centigrade scale, and 10° C. and —15° C. to the Fahrenheit scale.—*Camb.* 1923.

10. How would you test the accuracy of the fixed points of a mercury thermometer ? If a difference of 2·7 mm. in the barometric pressure causes a difference of a tenth of a degree Centigrade in the boiling point, what should a Fahrenheit thermometer read for the boiling point o water when the barometer reads 750 mm. ?—*Lond.* 1923.

11. A faulty Fahrenheit thermometer is handed to you for report as to the accuracy of its readings at the upper and lower fixed points. Describe, *in detail*, how you would conduct the necessary tests.

If the readings you obtained were 215° and 30° respectively, what reading on this instrument would record the correct temperature on the Fahrenheit scale?

What are the *distinctive features* about a sensitive mercury thermometer which reads to 1/10 degrees?—*Lond.* 1926.

12. Describe the clinical thermometer used for taking the temperature of the human body. Some clinical thermometers are guaranteed by the makers to attain the temperature of the body in half-a-minute; others require two minutes. What difference would you expect to find in the construction of the two instruments?—*Camb.* 1921.

13. Define the *coefficient of linear expansion* of a solid.

A barometer with a brass scale reads 29.72 inches on a day when the temperature is 59° F. If the scale graduations are correct at 0° C., what is the true height of the barometer at this time? [Coefficient of linear expansion of brass = 0.000019 per Centigrade degree.]—*Lond.* 1924.

14. What is meant by the "coefficient of linear expansion" of a solid? Describe one method of finding this coefficient.

The coefficient of *cubical* expansion of iron is given in the tables as lying between 0.000033 and 0.000044. Within what limits would you expect the coefficient of linear expansion of iron to lie? Give a reason.—*Scot.L.C.* 1926.

15. Explain how the disadvantages resulting from expansion are overcome in the case of watches and clocks.—*C.W.B.* 1923.

16. A man wishes to put an aluminium ring on a rod of iron of 1 inch diameter, but it is 0.001 of an inch too small in diameter. How much should its temperature be raised before it will just slip on? Subsequently he wishes to remove it again, but now he has to heat both metals together. Through how many degrees must this be done? [The coefficient of expansion of aluminium is 0.000025 and that of steel 0.000010.]—*M.L.L.S.B.*

17. Explain how a pendulum can be made whose effective length does not alter with temperature.

What is the length of the iron rod whose expansion would be equal to that of a brass rod 80 cms. long, if the two were heated through the same range of temperature? [Coefficient of expansion of brass = .000019, of iron = .000012.]—*Camb.* 1921.

Chapter II.

EXPANSION OF LIQUIDS AND GASES.

17. Expansion of Liquids. A little thought will show that the only coefficient of expansion we can consider here is the coefficient of *cubical* expansion. Further, a liquid must be contained in a vessel of some sort, and the problem is complicated by the fact that, on heating, the containing vessel will also expand, making the expansion of the liquid appear less than it really is.

The amount by which 1 c.c. of a liquid *really* expands when heated 1 degree Centigrade is known as the *coefficient of real or absolute expansion* of that liquid.

The amount by which 1 c.c. of a liquid *appears* to expand when heated 1 degree Centigrade is known as the *coefficient of apparent or relative expansion* of that liquid.

The first coefficient is often represented by the symbol C_r and the second by C_a. Evidently C_r is greater than C_a.

18. Proof that $C_r = C_a + C_g$. Suppose a liquid whose coefficient of real expansion is C_r and whose coefficient of apparent expansion is C_a, is contained in a vessel whose coefficient of expansion (cubical, of course) is C_g.

Consider 1 c.c. of the liquid, and let it be heated through 1° Centigrade.

Its volume will now be $1 + C_r$.

But the vessel will have expanded by an amount C_g.

$\therefore$ the volume of the liquid will *appear* to be only $(1 + C_r) - C_g$.

$\therefore$ 1 c.c. of the liquid will *appear* to have expanded by $C_r - C_g$, i.e., $C_a = C_r - C_g$ or $C_r = C_a + C_g$.

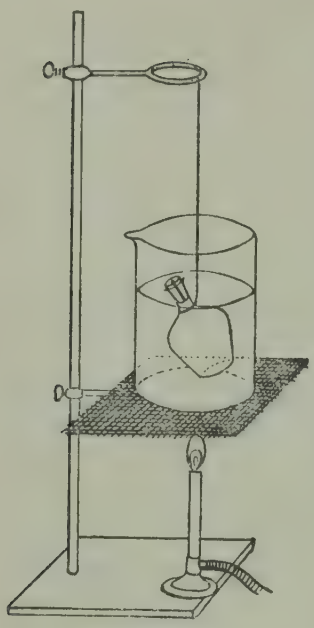

Fig. 12.
Coefficient of Apparent Expansion of a Liquid.

19. How C_a is found. A specific gravity bottle is weighed and then exactly filled with the liquid in question. (say glycerine). It is then weighed again and the temperature is noted.

The specific gravity bottle is then surrounded by water which is boiled (fig. 12).

The glycerine expands and some of it escapes. When this escape ceases, the bottle is taken out, allowed to cool and weighed once more. C_a is then worked out as in the following example :—

Weight of empty bottle, 23.49 gms.
Bottle and glycerine before expansion, 85.93 gms.
Bottle and glycerine after expansion, 83.37 gms.
Initial temperature, 16° C. Final temperature, 100° C.

Let s be the specific gravity of glycerine at 16° C., and C_a its coefficient of apparent expansion.

Weight of glycerine originally contained in bottle
$85.93 - 23.49 = 62.44$ gms.

Weight of glycerine after expansion $= 83.37 - 23.49 = 59.88$.

Now if the glycerine left in the bottle were heated once more to 100°, it would evidently fill the bottle exactly.

$\therefore \frac{59.88}{s}$ c.c. for a rise of temp. of 84 deg. become $\frac{62.44}{s}$ c.c.

$\therefore$ 59.88 c.c. for a rise of temp. of 84 deg. become 62.44 c.c.

$\therefore$ 59.88 c.c. for a rise of temp. of 84 deg. *expand* 2.56 c.c.

$\therefore$ 1 c.c. for 1 deg. expands $\frac{2.56}{59.88 \times 84} = .00051$ c.c.

i.e., $C_a = .00051$.

20. How C_r is found. We have found C_a, but what about C_r? From the relation $C_r = C_a + C_g$, we see that to find C_r we must have C_g as well as C_a.

It looks at first sight as though we might find the coefficient of *linear* expansion of glass by the method of

§12, and then multiply by 3, so obtaining C_g (§9). Unfortunately we have no guarantee that, after such violent treatment as melting and blowing, the coefficient of glass would still be the same. The method adopted is as follows:—

Taking the same bottle as was used for the glycerine experiment, we use it to find C_a for *mercury*. Now C_r for *mercury* has been obtained very accurately by a method which will be considered presently (§21). Since we now know C_a and C_r for mercury, we can find C_g by subtraction, and we can then use this to find C_r for glycerine.

21. How C_r for Mercury was found.

Let V_o = volume of a certain quantity of mercury at 0° C.
and V_t = its volume at $t°$ C.
Then $V_t = V_o(1 + C_r t)$ . . . . (§10).
Now if D_o = density of mercury at 0°,
And D_t = its density at $t°$,
We have mass of mercury at $0° = D_o V_o$.
And mass of mercury at $t° = D_t V_t$.
But the mass has not changed.
$\therefore D_o V_o = D_t V_t$.

$$\therefore \frac{V_t}{V_o} = \frac{D_o}{D_t}$$

But $\frac{V_t}{V_o} = 1 + C_r t$ [for $V_t = V_o(1 + C_r t)$].

$$\therefore \frac{D_o}{D_t} = 1 + C_r t.$$

If then we can find the *ratio* of the density of mercury at 0° to its density at some other known temperature $t°$, we shall have all the quantities necessary for finding C_r.

The experimental details for the determination of the ratio $\frac{D_o}{D_t}$ are rather complicated, and will not be discussed here, but the underlying principle is simple enough.

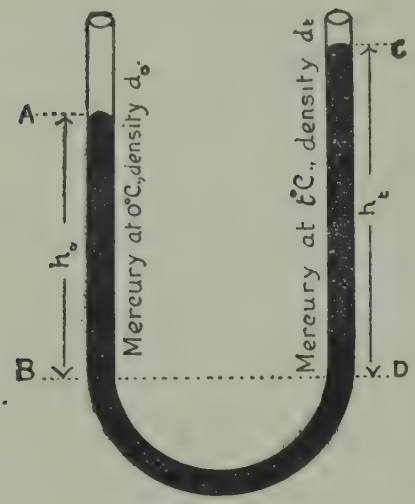

Fig. 13.
Absolute Expansion of Mercury.

Suppose (fig. 13) we have a U-tube containing mercury, and that by surrounding the two limbs with suitable jackets we maintain one of them, AB, at a temperature of 0° C., and the other, CD, at some other known temperature $t°$. Now it is shown in elementary hydrostatics that

$$\frac{D_o}{D_t} = \frac{\text{height of column CD}}{\text{height of column AB}} = \frac{h_t}{h_o}$$

and that *this ratio is quite independent of the bore of the tube.* Thus the fact that the bore of the tube increases with rise of temperature will not affect the result.

Hence we have the value of the ratio $\frac{D_o}{D_t}$ and this $= 1 + C_r t$. But t is known, and therefore C_r is readily found.

Example 8. If a glass weight thermometer contains 30 gms. of mercury at 0° C., and 29.529 gms. at 100° C., find the coefficient of cubical expansion of the glass. [The coefficient of absolute expansion of mercury is .000182].

If s be the specific gravity of mercury at 0° C., we have, working as in the glycerine example of §19,

$\frac{29.529}{s}$ c.c. for a rise of 100° C. become $\frac{30}{s}$ c.c.

$\therefore$ (multiplying by s)

29.529 c.c. for a rise of 100° C. become 30 c.c.

$\therefore$ 29.529 c.c. for a rise of 100° C. *expand* .471 c.c.

$\therefore$ 1 c.c. for a rise of 1° C. expands $\frac{.471}{29.529 \times 100}$

$= .000159$ c.c.

i.e., C_a for mercury $= .000159$,

but C_r for mercury $= .000182$.

$\therefore$ $C_g = C_r - C_a$ for mercury $= .000182 - .000159$

$= \mathbf{.000023.}$

Example 9. Suppose that, using the glass weight thermometer of Example 8, the coefficient of relative expansion of glycerine were found to be .000510, what would be its coefficient of real expansion ?

For this particular thermometer, $C_g = .000023$.
$\therefore\ C_r = C_a + C_g = .000510 + .000023$
$= \mathbf{.000533.}$

Example 10. The density of mercury at 0° C. is 13.596 gms. per c.c., and at 100° C. is 13.352 gms. per c.c. ; its relative coefficient of expansion in a steel vessel is .000147. What is the coefficient of linear expansion of steel ?—*Lond.* 1922.

Using the formula, $D_o = D_t(1 + C_r t)$ (§21),
We have $D_o = 13.596$, $t = 100$, and D_t or $D_{100} = 13.352$.
$\therefore\ 13.596 = 13.352\ (1 + 100C_r)$.
$\therefore\ 13.596 = 13.352 + 1335.2C_r$.
This gives $C_r = .000183$.
Considering now its expansion in the steel vessel, we have $C_a = .000147$, $C_r = .000183$, and C_g (coefficient of cubical expansion of steel) is to be found.
From $C_r = C_a + C_g$,
We have $.000183 = .000147 + C_g$,
Whence $C_g = .000036$.
The coefficient of *linear* expansion, being one-third of this, is **.000012.**

Example 11. The coefficient of cubical expansion of mercury is 0.00018, and the coefficient of linear expansion of glass is 0.000009. What volume of mercury must be placed in a glass specific gravity bottle in order that the volume not occupied by the mercury shall be the same at all temperatures ?—*Lond.* 1924.

Let V = volume of bottle ;
Let M = volume of mercury.
Coefficient of *cubical* expansion of glass $= 3 \times 0.000009 = 0.000027$.
$\therefore$ if temperature is raised $T°$, volume of bottle increases by $0.000027 \times V \times T$,
and volume of mercury increases by $0.00018 \times M \times T$.
If volume of unoccupied part is to remain the same, these quantities must be equal.
$\therefore\ 0.000027 \times V \times T = 0.00018 \times M \times T$.

T cancels, and we get $\dfrac{M}{V} = \dfrac{0.000027}{.00018} = \dfrac{3}{20}$

Thus the volume of the mercury must be 3/20, or roughly, one-seventh of the volume of the bottle.

22. Comparison of Different Liquids. The fact that different liquids have different coefficients of expansion

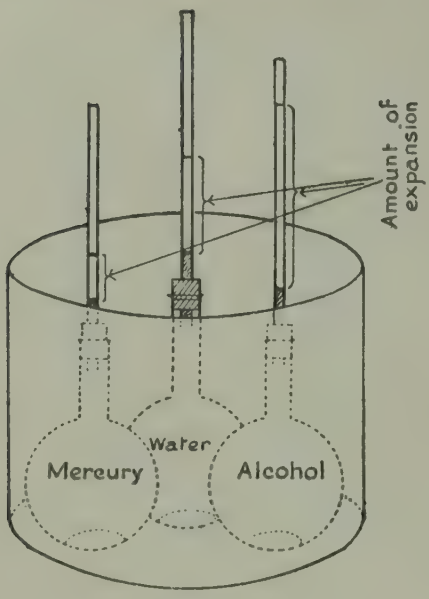

Fig. 14.
Unequal Expansion of Liquids.

may of course be shown by applying to each the experiment of §21. A rough comparison may be made, however, by taking a number of small flasks of equal size, one for each liquid. A cork and tube is then fitted into each as shown in fig. 14, the liquid being squeezed a little way up the tube. The flasks are then placed in a bath of water which is gradually heated, and the results are noted. If such an experiment is carried out, for instance, with mercury, water, and alcohol, it is found that alcohol expands most of the three and mercury the least.

As far as can be observed, for equal intervals of temperature (say 10° C.) the mercury expands by equal amounts, while in the other two cases the amounts increase with rise of temperature.

By a slight modification of the experiment, illustrated in fig. 15, we can study the *absolute* expansion of water (or other liquid).

The volume of the flask is measured and one-seventh of that volume of mercury is introduced (cf. Example 11, §21). The expansion of the flask is then compensated by that of the contained mercury. With this apparatus it is possible to show that water *contracts* as it is heated from 0° to 4° C., and expands beyond 4° C. (cf. next paragraph).

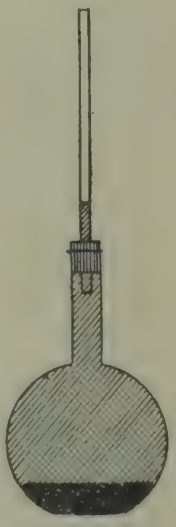

Fig. 15.
Real Expansion of Water.

23. Irregular Expansion of Water. This is well illustrated by **Hope's Experiment,** which is carried out with the apparatus shown in fig. 16. The experiment is begun with the water in the cylinder at about 12° C., while the trough round the middle of the cylinder is filled with a

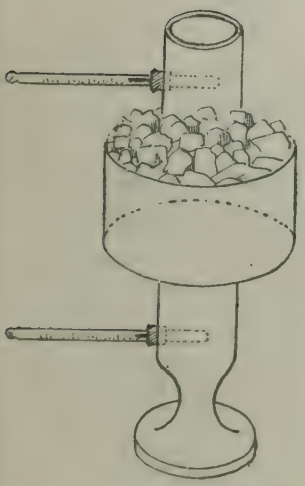

Fig. 16.
Hope's Experiment.

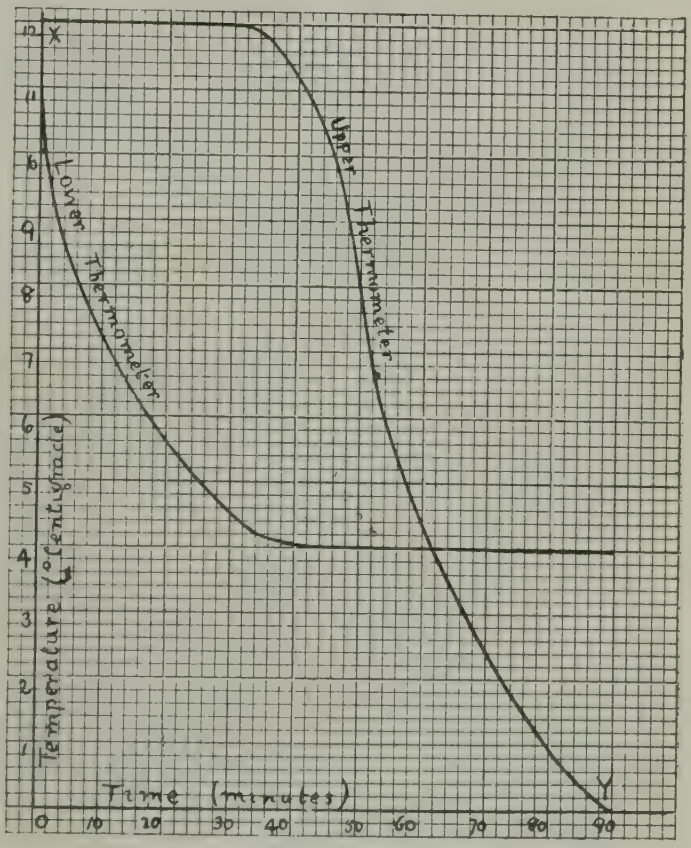

Fig. 17. Hope's Experiment.

freezing mixture. It is found that the temperature indicated by the lower thermometer falls steadily till a reading of 4° C. is reached, while the temperature of the upper part of the liquid remains constan tat about 12° C.

After this stage is reached, the reading of the *upper* thermometer falls steadily until a reading of 0° C. is attained, after which ice begins to appear on the surface.

The lower thermometer continues to record 4° C. The results may be conveniently illustrated by a graph (fig. 17).* The experiment shows that water at 4° C. is denser than at any other temperature attained in the experiment.

But if 4° C. is the temperature of *maximum density* of water, it must also be the temperature of *minimum volume*. Therefore water at a temperature below 4° C. will contract on *heating* until it reaches this temperature of minimum volume, while water at a temperature above 4° C. will contract on *cooling* until this same temperature is reached. In fact, a quantity of water at 8° C. has very nearly the same volume as at 0° C.

Important consequences in Nature follow from the fact that the temperature of maximum density of water is considerably above the freezing point. As things are at present, ice forms on the surface while the water in the lower layers is still at 4° C. This warmer water cannot lose much heat by conduction (water being a very bad conductor), and it cannot lose it by convection, because to do so it would have to *rise* to the colder region at the top, and it cannot rise because it is denser than the surface water. Thus it never reaches the freezing point, and even in severe winters the thickness of ice on deep lakes, etc., is never very great.

If water did not behave in the curious way mentioned, the water of our deep lakes would become frozen in the winter into a solid mass of ice so thick, that even during the summer only its upper layers would be melted.

24. Expansion of Gases. It will be well to consider first what happens when a gas at constant temperature is subjected to change of pressure. This case is covered by **Boyle's Law,** which states that, **If the temperature of a given mass of gas remains constant, then the volume is inversely proportional to the pressure.**

25. Coefficient of Expansion of a Gas. When measuring the expansion of a solid or liquid we could ignore

* Obtained experimentally by G. F. Westcott and O. B. Westcott (Hele's School, Exeter).

changes of pressure. Boyle's Law reminds us that in the case of a gas we must take care that the experiment is carried out *at constant pressure.*

26. Coefficient of Expansion of Air. One method of finding this is to take a piece of glass tubing about a foot long, clean and dry it, and introduce a pellet of mercury into it by suction. The mercury should be about 3 inches from one end, and the other end is then sealed up. The tube is then attached to a millimetre scale as shown in the figure, and introduced into a jar of water (not shown) which has been cooled to 0° C. by the addition of ice.

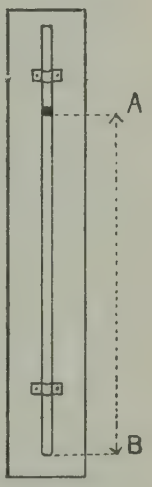

Fig. 18. Coefficient of Expansion of Air.

The length AB of the air column is now read off, after which some hot water is introduced into the jar. This causes the air to expand, and after thorough stirring, the length of the column is again observed, together with the temperature. Thus suppose :—

	Length of air column.	Temperature.
1st time.	22.4 cms.	0° C.
2nd time.	25.0 cms.	32° C.

If the tube is of uniform bore, volume is proportional to length.

$\therefore$ 22.4 vols. for 32 degs. have expanded by 2.6 vols

$\therefore$ 1 vol. for 1 deg. has expanded $\dfrac{2.6}{22.4 \times 32}$

$= .00363.$

If the experiment is carefully carried out, the value .00366, or 1/273, is obtained, and the result is the same *whatever gas is employed.* Hence we obtain *Charles' Law*, which states that, *If a certain mass of gas is heated at constant pressure, its volume increases for each rise of* 1° *C. by* 1/273 *of its volume at* 0° *C.*

27. If the gas is cooled instead of heated, then for each fall of 1° C. its volume *decreases* by 1/273 of its volume at 0° C. At —10° C. the volume would have diminished by 10/273 of the volume at 0° C., and, if the gas continued to contract at the same rate, at —273° C. the volume of the gas would have been reduced to nothing at all! (As a matter of fact, every gas becomes liquid before such a low temperature is reached.)

The temperature (—273° C.) at which the volume of a gas would become zero if it continued to obey Charles' Law, is known as the Absolute Zero.

28. Absolute Temperature. It is often convenient, as we shall see shortly, to reckon temperatures as being so many degrees above the *absolute zero* (—273° C.) instead of as being so many degrees above the freezing point of water. Temperatures so reckoned are known as **Absolute Temperatures,** and are said to be measured on the *Absolute Scale.*

Thus 10° C. is clearly 273° + 10° or 283° above absolute zero, so we say 10° C. = 283° Absolute. Evidently to bring degrees Centigrade to degrees Absolute, we have only to add 273.

29. Volume Proportional to Absolute Temperature. Applying the formula of §10 to a gas, and noting that $c = \frac{1}{273}$, we have :—

$$V_t = V_0 (1 + \tfrac{1}{273} t).$$

Also $V_{t'} = V_0 (1 + \tfrac{1}{273} t')$.

$$\therefore \frac{V_t}{V_{t'}} = \frac{V_0 (1 + \frac{1}{273} t)}{V_0 (1 + \frac{1}{273} t')}$$

$$= \frac{273 + t}{273 + t'}$$

But $273 + t = T$, where T is the corresponding *absolute* temperature. Similarly $273 + t' = T'$.

Hence we have $\frac{V_t}{V_{t'}} = \frac{T}{T'}$

Charles' Law may therefore be stated in the following form, which the student will find much easier to commit to memory than the one already given.

If a given mass of gas is kept at constant pressure, then its volume varies directly as its absolute temperature.

30. Mathematical Expression. Both Boyle's Law and Charles' Law are included in the equation $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

For if a gas is kept at constant temperature, T_1 and T_2 cancel, and we have $P_1\ V_1 = P_2\ V_2$, which is Boyle's Law; while if the gas is kept at constant pressure, P_1 and P_2 cancel, and we have $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ or $\frac{V_1}{V_2} = \frac{T_1}{T_2}$ which is Charles' Law.

31. Different Gases. It has already been stated that the Laws of Boyle and Charles apply to *all* gases. This fact may be shown by working the experiments described above with a number of different gases. Or (to verify the fact so far as Charles' Law is concerned) we may take two small flasks of equal size and fill them with two different gases, say oxygen and nitrogen. The flasks are filled with delivery tubes so arranged that any gas expelled may be collected in a graduated collecting tube. The flasks are then raised to the same temperature by surrounding them with the same bath of hot water, when it is found that the same volume of gas is expelled from each. (To avoid confusion, the clamps holding the collecting tubes in position are not shown in the figure.)

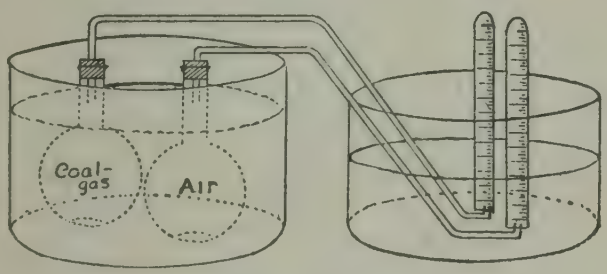

Fig. 19.
Equal Expansion of different Gases.

Solids and liquids, unlike gases, differ very widely in their coefficients of expansion.

32. Coefficient of Increase of Pressure. If a quantity of gas is enclosed so that expansion is impossible, and we raise its temperature, the *pressure* increases. The exact amount is easily seen from the equation $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

For if the volume remains constant (i.e., if $V_1 = V_2$), then the "V's" cancel, and we get $\frac{P_1}{T_1} = \frac{P_2}{T_2}$ or $\frac{P_1}{P_2} = \frac{T_1}{T_2}$

i.e., **if a mass of gas is kept at constant volume, then its pressure varies directly as its absolute temperature.**

To verify this experimentally, we may use the apparatus of fig. 20. The air (or other gas) is enclosed in a dry bulb, A, connected with a tube BC, of small bore, bent twice at right angles. At C this tube joins on to a rubber tube, CDE, which is in turn connected with a reservoir F, containing mercury. F can be raised or lowered by means of a string passed over a small pulley, and the level of the mercury in the reservoir may be recorded by means of a vertical metre scale.

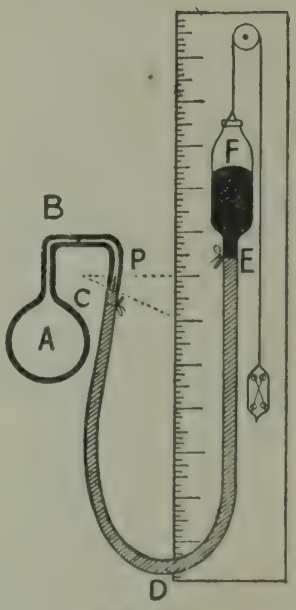

Fig. 20.
Constant Volume Air Thermometer.

To begin with, A is surrounded by a bowl of melting ice, and when the air has had time to take the temperature 0° C., the reservoir is moved until the mercury columns in the two sides of the tube are at the same level. Suppose the contained air reaches as far as P. This point is marked with a file scratch or gummed paper. The barometer is now read, and this gives the pressure of the air contained in A. Suppose this is 75.2 cms. (As the tube BC is of small bore, the fact

that the air in it lies beyond the containing ice jacket will not make an appreciable difference.)

The ice is now exchanged for water, the temperature of which is observed; say this is 20° C. When A has had time to acquire this temperature, the reservoir is raised until the mercury on the left is once more at the point P. The *difference* between the heights of the mercury on the left and right is now observed—suppose this difference is 5.5 cms. The pressure of the air in A is therefore $5.5 + 75.2 = 80.7$ cms.

We now have two sets of readings.

At 0° C. or 273° Abs. the pressure was 75.2 cms.

At 20° C. or 293° Abs. the pressure was 80.7 cms.

Now $\frac{293}{273} = 1.073$ and $\frac{80.7}{75.2} = 1.073$.

Thus as far as we have gone, the pressure is found to be proportional to the absolute temperature. Further sets of readings would be obtained, and the same relationship would be found to hold good. The results of this experiment may also be used to show that *if a certain mass of gas is heated at constant volume, its pressure increases for each rise of* 1° *C. by* 1/273 *of its pressure at* 0° *C.* (cf. §26).

This is simply an alternative way of stating the law, just as we had two ways of stating Charles' Law. The fractional increase of pressure just referred to (1/273) is called *the coefficient of increase of pressure at constant volume.*

33. Air Thermometers. The apparatus described in §26 could evidently be used as an air thermometer. Thus suppose that when surrounded by melting ice, the length of the air column was 22.4 cms. We mark this point 0° C. Where must we put the mark for (say) 50° C. ?

Now 0° C. = 273° Abs.

And 50° C. = 273 + 50 = 323° Abs.

Vol. at 273° Abs. is represented by number 22.4

$\therefore$ Vol. at 323° Abs. is represented by number $\frac{22.4}{273} \times 323$

$= 26.5.$

Thus we should measure off 26.5 cms. from the bottom of the tube, and mark the point 50°. In a similar way we could mark off all the temperatures we required. But such a thermometer would suffer from two disadvantages.

(i.) Suppose we were taking the temperature of a *small* quantity of a substance (water, etc.), the greater part of the air column would not be surrounded with the substance at all.

(ii.) Our graduations would only be correct *for the pressure at which they were made.* If the atmospheric pressure increased, for instance, all our markings would be too high up on the scale. Such an instrument would be called a *Constant Pressure Air Thermometer*, and would not be of much use.

34. The Constant Volume Air Thermometer is a much more useful instrument. It consists of the apparatus described in §32. We can obtain any desired pressure by raising or lowering the reservoir F. Now (taking the numbers used in §32) when the pressure is adjusted to 75.2 cms.,* and A is at 0° C., the mercury will be exactly at *P*. Suppose we wish to find the temperature of a quantity of hot water. A is surrounded with the water and the reservoir adjusted until the mercury is forced back to P. Suppose (allowing for the height of the barometer) the pressure in A is found to be 92.4 cms., and that the required *absolute* temperature is *T*.

Then at 273° A (= 0° C.) the pressure was 75.2
and at *T*° Abs. the pressure was 92.4.

But at constant volume, pressure is proportional to absolute temperature.

$$\therefore \frac{T}{273} = \frac{92.4}{75.2}$$

Working this out, we obtain $T = 336$.

∴ the Centigrade temperature is 336—273 = 63° C.

*[Q.—How would this be done, supposing the reading of the barometer to have been taken ?]

Example 12. A cylinder will burst at a pressure of 105 lbs per sq. in. Air is put into the cylinder at a pressure of 35 lbs

per sq. in., and the cylinder closed at 0° C. At what temperature will the cylinder burst?—*Camb.* 1923.

At constant volume, pressure is proportional to absolute temperature, or $\frac{P_1}{P_2} = \frac{T_1}{T_2}$

Here $P_1 = 35$, $T_1(0° \text{ C.}) = 273°$ Abs., $P_2 = 105$, $T_2 = ?$

Substituting, we have $\frac{35}{105} = \frac{273}{T_2}$

$\therefore$ $T_2 = 819°$ Abs. $= (819—273)°$ C. $=$ **546° C.**

Example 13. A flask containing 300 c.c. of air at 100° C., is closed, and then opened under water. It is found that 70 c.c. of water enter the flask, the temperature now being 13° C. From these figures find a value for the coefficient of expansion of air.—*Lond.* 1925.

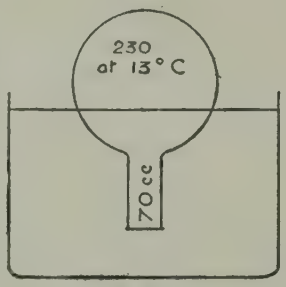

Fig. 21.
Coeff. of Expansion of Air.

If the air (230 c.c.) at 13° C. were heated once more to 100° C., it would evidently fill the flask.

$\therefore$ 230 c.c. heated 87 deg. expand 70 c.c.

$\therefore$ 1 c.c. heated 1 deg. expands $\frac{70}{230 \times 87}$

$=$ **.00350** or about $\frac{1}{286}$

(N.B.—The experimental method indicated here should be carefully noted, as it is often used for finding an approximate value for the coefficient of expansion of a gas.)

QUESTIONS ON CHAPTER II.

1. If the coefficient of relative expansion of mercury in a glass weight thermometer is found to be .000159, and the coefficient of absolute expansion of the mercury is .000182, find the coefficient of cubical expansion of the glass.

2. Using the weight thermometer of Question 1, it was found that the coefficient of relative expansion of sulphuric acid was .000466. Find the coefficient of absolute expansion of the acid.

3. A mercury column 32.4 cms. high at 0° C., balances a column 33.0 cms. high at 100° C. What value does this give for the coefficient of absolute expansion of mercury?

4. Calculate the absolute temperatures corresponding to 20° C 100° C., —40° C.

5. Calculate the absolute temperatures corresponding to 98° F. 62° F., —40° F.

6. 1 gm. of hydrogen occupies 11.2 litres at 0° C. and 76 cms. pressure. What volume would it occupy, (i.) at 60° C. and 76 cms. pressure, (ii.) at 0° C. and 114 cms. pressure ?

7. A certain quantity of air occupies 580 c.c. at 17° C. and 74 cms. pressure. What would be its volume at N.T.P. ?

8. The volume occupied by 16 gms. of oxygen at N.T.P. is 22.4 litres. What would be its volume at 15° C. and 75 cms. pressure ?

9. A uniform glass tube contains a column of air separated from the outer air by a pellet of mercury (cf. §26). At 0° C. the column measures 14.3 inches, and at 40° C., 16.4 inches, the pressure remaining constant. What value does this give for the coefficient of expansion of air ?

10. The air in a Constant Volume air thermometer was found to exert a pressure of 76 cms. when the bulb was surrounded by melting ice, and of 95 cms. when it was surrounded by water at 67° C. What value does this give for the coefficient of increase of pressure at constant volume ?

11. A Constant Volume air thermometer showed a pressure of 103.8 cms. of mercury when the bulb was surrounded by boiling water, and a pressure of 117.7 cms. when the water was replaced by some hot glycerine. Find the temperature of the glycerine.

12. A glass vessel just holds 1 kilogram of mercury at the temperature of 0° C. What weight of mercury will it hold at 100° C. ? [Coefficient of dilatation of mercury is .00018 ; coefficient of linear expansion of glass is .00001.]—*M.L.L.S.B.* 1921.

13. Distinguish between apparent and real expansion of a liquid or gas.

Describe how the apparent expansion of mercury in glass may be determined.—*O.C.* 1924.

14. What is meant by the *coefficient of real expansion*, and the *coefficient of apparent expansion* of a liquid ?

How are these two coefficients related to one another ?

The stem of a mercury thermometer has a cross-sectional area of 0.003 sq. cm. It is desired that the scale of the thermometer shall be 10 cms. long between the 0° C. and the 100° C. marks. By what volume must the mercury expand when heated through 100° C. ?

If the coefficient of apparent expansion of the mercury in glass is 0.00016, what must be the volume of the mercury in the thermometer at 0° C. ?—*Oxf.* 1924.

15. Explain the connection between a change in volume caused by heating a liquid and the corresponding change in density, and describe some experimental method in which one of these changes is measured by means of the other.—*Lond.* 1923.

16. Describe and explain how the coefficient of absolute expansion of a liquid is determined.
Which coefficient of expansion would you employ, the absolute or the relative, in correcting the height of the mercury column of a barometer for temperature? Give reasons.—*Camb.* 1925.

17. What experiment would you make to study the change in the volume of a given quantity of water as the temperature is raised from 0° C. to 15° C.? Describe the results you would expect to obtain, and explain the importance of their bearing on the freezing of a pond.—*Lond.* 1921.

18. Describe what takes place when a lake freezes over. Your description should make it clear why very deep lakes in this country seldom freeze over. Mention any laboratory experiment supporting your explanation of this fact.—*Scot.L.C.* 1926.

19. A spherical glass bottle, whose capacity is 1,000 c.c. at 0° C., is left for a time unstoppered in a room where the temperature is 15° C., and the mercury barometer stands at 755 mm. The stopper is inserted and the bottle is then immersed in a cooling mixture at —10° C. Find the pressure of the air in the bottle, assuming that no air passes the stopper during the cooling. (The contraction of the glass may be neglected.)—*M.L.L.S.B.* 1922.

20. Explain carefully what is meant by a *rise of temperature of* 1° *C.* The pressure in a constant volume air thermometer is 770 m.m. of mercury at 15° C.; what will it be at 20° C.? —*Oxf.* 1925.

21. Describe an air thermometer, such as you could make for yourself. How would you calibrate it? State the law or laws upon which its usefulness depends.—*M.L.L.S.B.* 1923.

22. Describe the constant volume air thermometer, and explain how you could use it to measure the temperature of the laboratory, no other thermometer being provided.—*Camb.* 1921.

Chapter III.
CHANGE OF STATE.

35. States of Matter. Matter may exist in three forms—solid, liquid, and gas. These are known as the three *states* of matter.

36. Melting of Ice. Suppose on a very cold winter's day in the open, we put a quantity of clean broken ice into a beaker, and take the temperature. It might be, perhaps, —10° C.

On gently warming and stirring, the temperature rises until 0° C. is reached.

There would be a slight increase of volume, though in the given circumstances this would not be observed.

On continuing to heat, *the temperature would remain constant at 0° C. until all the ice was melted.* The ice would be floating on the water. Therefore the density of ice is less than that of water, and so during the change from ice to water, *contraction must have taken place* (the amount is actually about 10 %). When the ice is completely melted, the temperature begins to rise more and more until the boiling point is reached. The volume continues to go less, however, until 4° C. is reached (§23), after which expansion once more takes place.

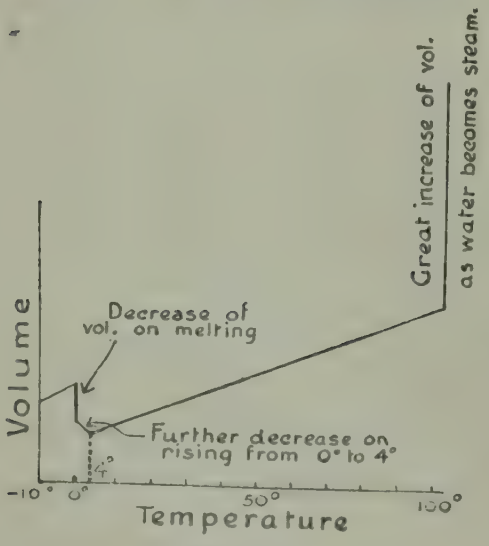

Fig. 22a.
Change of Volume with Temperature.

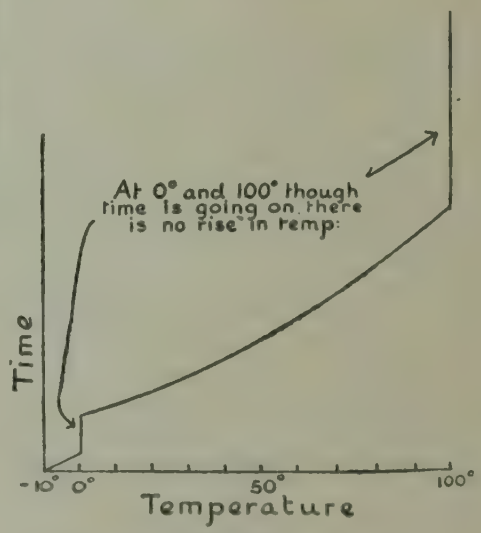

Fig. 22b.
Change of Temperature with Time.

The results may be completely illustrated by rough graphs, one showing how the volume changes with rise of temperature (fig. 22a), the other showing the *time* required to bring about these changes (fig. 22b). In fig. 22b the gradually increasing steepness of the slope is due to the fact that as the water becomes hotter it loses heat to the surrounding air at a greater rate. Hence it takes longer to heat it say from 70° to 80°. than from 20° to 30°. The graphs, however, are not quantitative, and are only meant to indicate the general character of the changes.

37. Contraction of Ice on Melting. The fact that ice contracts on melting has been deduced from the observation that it floats on water. It may be shown by a more direct method, however. A quantity of paraffin oil contained in a test-tube is surrounded by a freezing mixture until its temperature is well below 0° C. It is now poured into a dry burette, and as soon as the temperature has reached 0° C. the burette reading is taken. Call this x.

Small pieces of dry ice are at once introduced. Ice is denser than paraffin, so it sinks in this liquid. Suppose the burette reading is now y, so that the volume of the ice is $y-x$. As soon as the ice has been completely melted the burette reading (z) is taken, and this will be found to be less than y, showing that the ice has contracted on melting.

[Q.—What is the *percentage* contraction? What is the specific gravity of the ice?]

38. Why does the Temperature not Rise? When we apply heat to a substance, we expect to find a rise of temperature. Why does this not occur when ice or other solid substance is being melted?

A substance is made up of minute particles called *molecules*, which are closely packed in the case of a solid, and not so closely packed in the case of a liquid. Further, these molecules are in a state of vibration, any increase in the average frequency of vibration being accompanied by an increase of temperature, and vice versa.

Now heat is a form of energy (§72), and when heat is supplied to a body it may either, (i.) increase the average kinetic energy of the molecules, i.e., increase the average frequency of vibration, so producing a rise of temperature ; or,

(ii.) cause the molecules to be dragged further apart, overcoming the force with which they attract one another. This will cause the solid substance to become liquid, but if the molecules are still vibrating with the same average frequency, there will be no rise in temperature. Heat which is thus expended in producing change of state instead of change of temperature is said to be *latent.*

39. Determination of Melting Point.

First Method. Melt some of the substance (e.g., naphthalene) in a wide test-tube, and support a thermometer so that its bulb is almost in the middle of the liquid. Now observe the temperature every half-minute, and plot the results on squared paper as in fig. 24.* One portion of the graph is horizontal, i.e., there is no fall of temperature as the time goes on, and this steady temperature is the melting point.

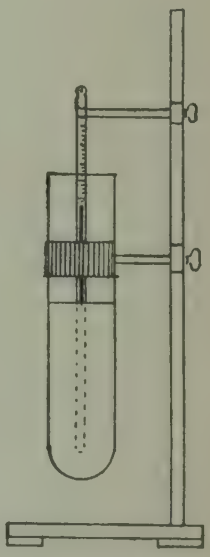

Fig. 23.
Obtaining a Cooling Curve.

The temperature does not fall, because the loss of heat to the surrounding air is being made up by the latent heat given out as the liquid substance becomes solid.

Note that in this experiment we have evidence that as liquid naphthalene solidifies it gives out heat · similarly for other substances.

Second Method. A piece of glass tubing is drawn out to a thickness of about 1 mm., and some of the powdered solid substance is introduced, occupying say half-an-inch in the tube. The tube is next fastened to a thermometer

*Obtained experimentally by R. Clinnick and K. McLennan (Hele's School, Exeter).

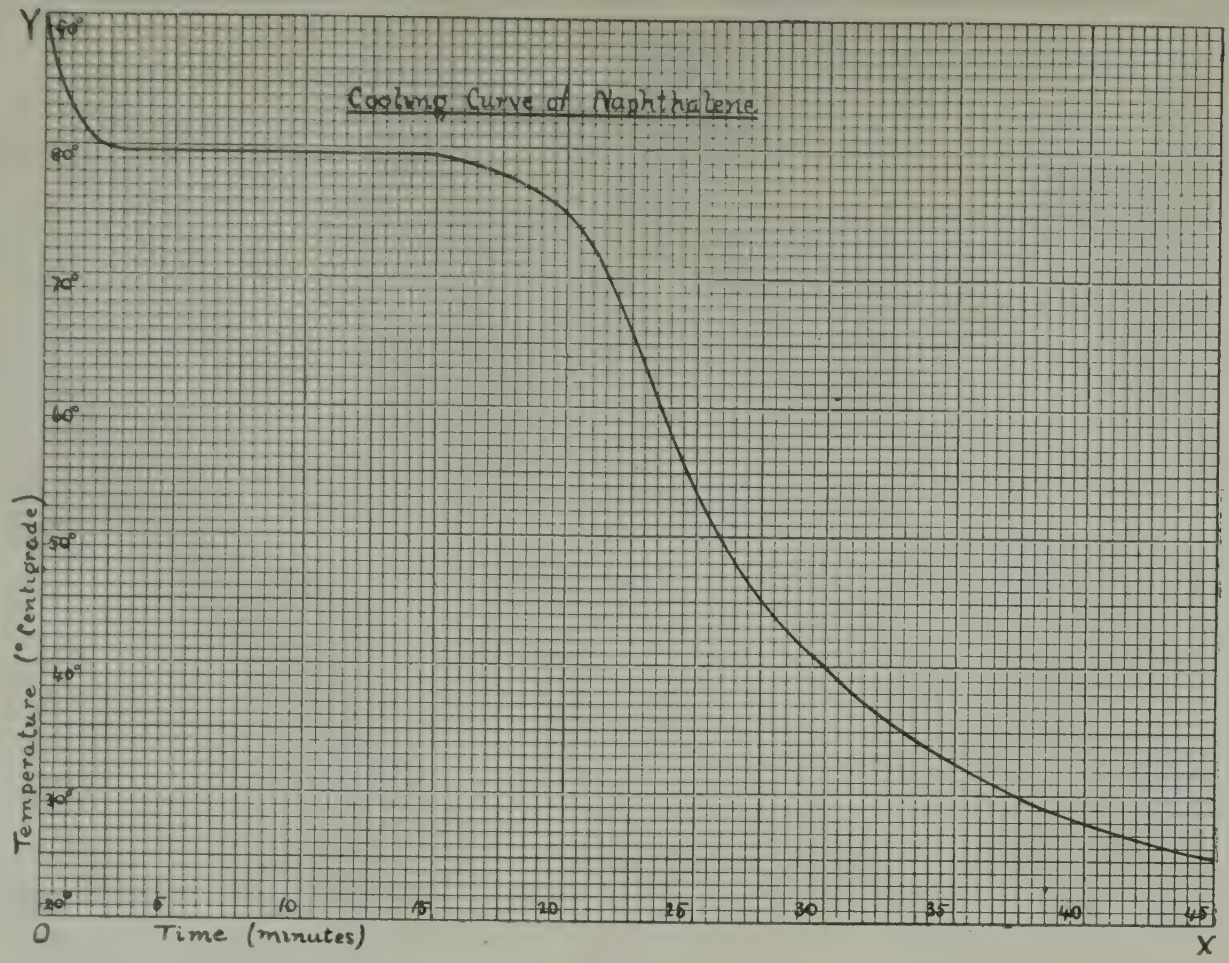

Fig. 24.

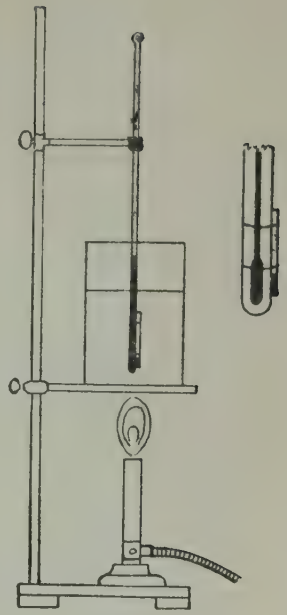

Fig. 25.
Determination of Melting Point.

with two rubber bands, the substance being just opposite the bulb. The whole is now placed in a beaker of water (as in fig. 25), which is heated. When the substance melts it suddenly becomes transparent, and the temperature at which the change takes place is noted.

40. Effect of Pressure on Melting Point. Ice under pressure melts at a lower temperature than 0° C. Thus if a block of ice is suitably supported, and a thin wire, to which two heavy weights are attached, is placed across it, the wire will cut its way right through the ice, but at the end of the experiment *the ice will still be in one piece.*

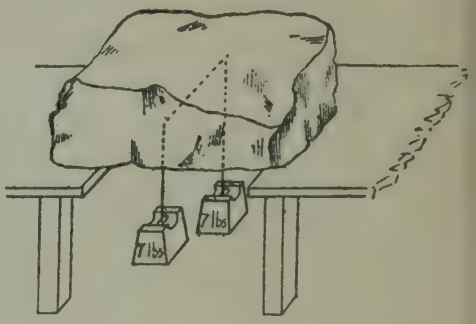

Fig. 26. Regelation.

The phenomenon is known as *regelation.*

The explanation is that just below the wire, where the ice is subjected to heavy pressure, the melting point is lowered. Suppose the new m.p. is —0.2° C. (the actual number is dependent on the pressure). The temperature of the mass of slowly melting ice is 0° C. Therefore the ice just below the wire, being at 0° C., is *above* its (new) melting point, and so it becomes liquid and flows over the wire. It is thus released from pressure, and freezes again because as the pressure becomes normal so does the melting point.

[Q.—Can you explain how a glacier makes its way past obstacles such as projecting rocks? Why snow forms icy lumps on one's boots in the course of a walk? Why it is impossible to make good snowballs when the temperature is much below freezing point?]

41. Evaporation and Boiling. Evaporation is a very familiar process. It takes place *at all temperatures*, but more readily at high temperatures than at low ones. Boiling, however, for any given liquid at a given pressure, takes place *at a fixed temperature.* Another contrast is

that evaporation takes place *only from the surface of the liquid*, while boiling takes place *throughout the mass of the liquid*.

42. Latent Heat of Vaporisation. This will be dealt with more fully in the next chapter. For the present we may notice that to turn a quantity of liquid into vapour *at the same temperature*, heat must be supplied, and this heat is known as *the latent heat of vaporisation*. The commonest case is seen in connection with the boiling of water, the temperature of the water keeping constant at 100° C., in spite of the fact that heat is being continually supplied. The explanation is quite similar to that given in §38.

43. Vapour Pressure. Suppose A and B (fig. 27) are two mercury barometers, the mercury standing at G and H respectively. By means of a curved pipette, let a single drop of water be introduced into B. It quickly vaporises, and the mercury is pushed from H to K. Clearly the water vapour is exerting a pressure which is measured by the length HK.

Now let another drop be introduced. The mercury column is pushed down further to the point L, and another drop might result in a further drop to the point M. (For clearness, the distances HK, KL, etc., are greatly exaggerated in the figure.)

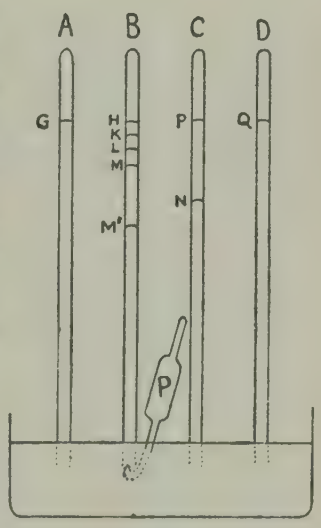

Fig. 27.
Vapour Pressure.

A point is soon reached, however, when the introduction of an additional drop of water is not followed by evaporation, and the mercury is not pushed down any further. The space is now said to be *saturated* with aqueous vapour, and the pressure exerted (measured by

the distance through which the mercury has been pushed downwards) is the *maximum pressure of aqueous vapour* for that particular temperature.

Before this final stage was reached, the space, though it contained aqueous vapour, was unsaturated; and though the vapour exerted a pressure, this pressure was evidently less than the *maximum* pressure just discussed.

Suppose, for instance, that the height of the barometer was originally 75.8 cms., and that when the space was saturated with water vapour at 15° C. the height was reduced to 74.5 cms., we should say that "the maximum pressure of aqueous vapour at 15° C. is 1·3 cms."

44. How to tell if a Vapour is Saturated. The last paragraph suggests a means of determining whether a vapour is saturated or not. We introduce a single drop of the corresponding liquid. If this gradually disappears the vapour was unsaturated. If it does not it must have been saturated.

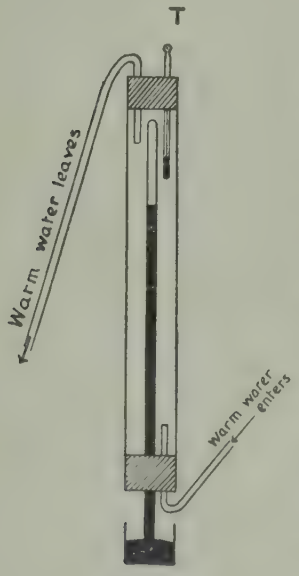

Fig. 28.
Vapour Pressure at different Temperatures.

45. Effect of Temperature. If a little more water is passed into the tube at this stage, it fails to evaporate, as we have seen. It remains floating on the top of the mercury. If, however, the temperature of the tube is raised (preferably by surrounding it with a water jacket of known temperature, as in fig. 28), more water *does* evaporate, and the mercury is pushed further and further down until a new maximum is reached at M′ (fig. 27). Thus if the new temperature is 50° C. we should find that HM′ would measure 9.1 cms., this being "the maximum pressure of aqueous vapour at 50° C."

Let us now go back to room temperature conditions, 15° C., by removing the water jacket. Part of the vapour inside B now *condenses*, the pressure is reduced, and the mercury rises from M′ to M. Clearly *the maximum vapour pressure increases with the temperature.*

46. Effect of Changing the Liquid. In fig. 27 C and D are two other barometer tubes, the mercury standing at P and Q respectively Let successive drops of *alcohol* be introduced into C. We shall find similar results to those obtained with the water introduced into B, except that the maximum pressure obtained at any given temperature will be greater. Thus at 15° the mercury might be pushed down to the point N. Using ether in the fourth tube, we should find a still greater maximum pressure for a given temperature. *At a given temperature the maximum vapour pressure depends on the liquid.*

47. Vapour Pressure and Boiling Point. Lastly, by surrounding B, C and D with suitable water (or steam) jackets, let the temperature be gradually raised, taking care that there is always a little surplus liquid floating on the top of the mercury as a guarantee that the space is saturated. At a certain temperature (different for each liquid) it will be found that the mercury is pushed down so as to be level with the mercury in the bowl. At this stage the vapour inside the tube is evidently exerting a pressure equal to that of the atmosphere. This will be found to happen when :—

B has been raised to 100° (boiling point of water).
C has been raised to 78° (boiling point of alcohol).
D has been raised to 35° (boiling point of ether).

Thus *when a liquid is at its boiling point, the maximum pressure of its vapour is equal to that of the atmosphere.*

From this we may obtain a useful definition of boiling point. **The boiling point of a liquid is that temperature at which the maximum pressure of its vapour is equal to that of the atmosphere.**

It will now be clear that the boiling point of water will be lower on days when the pressure is abnormally low, and vice versa.

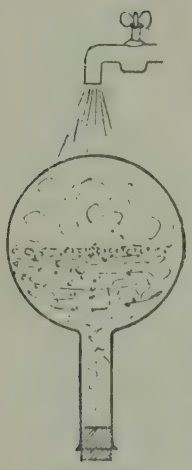

Fig. 29.
Water Boiling under Reduced Pressure.

48. Boiling under Reduced Pressure. It is easy to show by a direct method that under reduced pressure water boils below 100° C. Some *warm* water (at say 30° C.) is placed in a beaker under the receiver of an air pump. On working the pump the water soon begins to boil.

Another well-known experiment is to boil some water in a round-bottomed flask. While boiling, a rubber stopper is inserted and the bunsen burner is immediately withdrawn. The flask is now inverted and some cold water from the tap is run over it (fig. 29). The water begins to boil freely once more.

[Q.—Why ?]

49. Determination of Boiling Point. The principle just stated is sometimes used to find the boiling point of a liquid of which only a few drops are available.

A **J**-tube, ABC, is completely filled with mercury except for a small portion at the end A, this last being filled up with the liquid under examination. By putting the thumb over the end A and inverting the tube, the liquid can be transferred to the end C. After this, mercury is cautiously poured out until it stands at a lower level in the open limb than in the closed one. The **J**-tube is now heated by means of water—or sulphuric acid if the boiling point in question is above 100° C.—and when the mercury stands at the same level in both limbs the temperature is noted. This is the required boiling point.

Suppose, for instance, that alcohol had been placed in the end C. The two mercury columns would be at the same level when the thermometer recorded a temperature of 78° C.

[Q.—How could this method be adapted, (i.) to show that water boils when its vapour pressure is equal to that of the atmosphere, (ii.) to find the vapour pressure of aqueous vapour at temperatures above 100° C. ?]

The ordinary method of finding a boiling point, however, is to put the liquid in a distilling flask and boil it, a thermometer being in position with its bulb just below

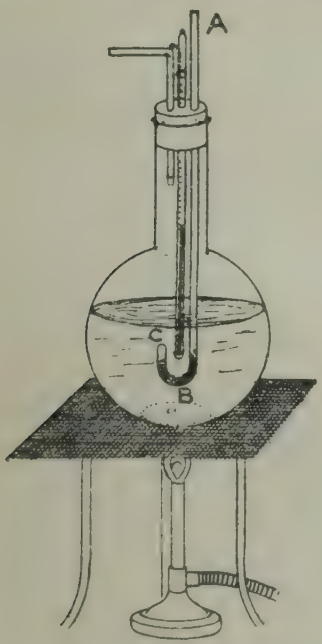

Fig. 30a.
Special Method of determining Boiling Point.

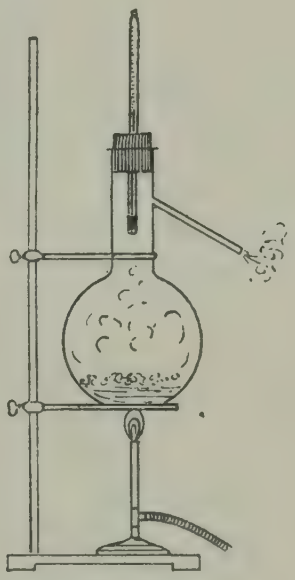

Fig. 30b.
Ordinary Method of determining Boiling Point.

the end of the side tube. By this method we eliminate any error due to solid impurities being dissolved in the liquid. If it is desired to recover the liquid, the side tube is connected with a Liebig's condenser.

Suppose we were dealing with a solution (of common salt for instance). It is instructive to notice that with the thermometer in the position shown in fig. 30b, we obtain the boiling point of the *solvent*—in this case water. If we wish to obtain the b.p. of the *solution*, we must allow the thermometer to dip into the liquid.

50. Vapour Pressure at various Temperatures. Reference to this has already been made (§45). The barometer tube containing drops of the liquid under examination (usually water) is surrounded with a wider tube as shown

in fig. 28. Warm water is poured into the wide tube or (better) is allowed to circulate through it from a geyser. The temperature is recorded by the thermometer T. The barometer tube is graduated in centimetres and millimetres, and the vapour pressure for any given temperature is readily obtained by subtracting the observed height from the original one.

51. Relative Humidity. The air always contains more or less water vapour, but is seldom saturated with it. ***The relative humidity of the atmosphere*** **is the ratio of the amount of water vapour actually present to the amount that would be required to saturate it at the existing temperature.**

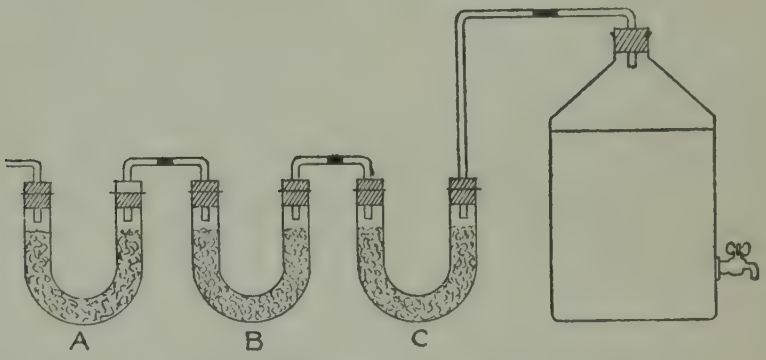

Fig. 31a. Chemical Hygrometer.

In connection with weather observations, it is often necessary to find the relative humidity. One method (very tedious in practice) is by the use of the *chemical hygrometer*. Several U-tubes, A, B, C, containing calcium chloride (or pumice stone soaked with concentrated sulphuric acid) are connected with a large aspirator containing water. The U-tubes are weighed, and then by turning on the water a slow current of air is drawn through them.

The volume of air drawn through the tubes is found by measuring the water run out from the aspirator, and the weight of water vapour contained in this air is indicated by the increase in the weight of the U-tubes. Dividing the weight of the water vapour by the number

of litres, we evidently obtain the weight actually present per litre. Call this x.

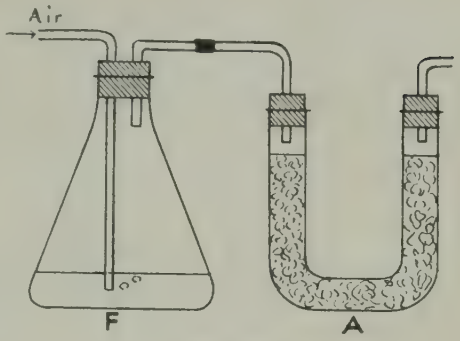

Fig. 31b.

To find the amount y that would be required to *saturate* 1 litre of the air, we could evidently arrange that before entering A, the air should bubble through water contained in a number of flasks such as F (fig. 31b). As a matter of fact, however, the amount of water vapour required to saturate 1 litre of air at various temperatures has been carefully determined "once for all" by various experimenters, and the results may be obtained from the tables that they have drawn up. The required relative humidity is thus equal to x/y.

A much shorter method in practice is to determine the ***dew point,*** i.e., **the temperature at which the water vapour actually present in the air would be sufficient to saturate it.** A metal cup (a bright nickel shaving tin does very well) is about two-thirds filled with water. Bits of ice are added from time to time, the liquid being constantly stirred with a thermometer. As soon as a mist begins to appear on the bright surface, the temperature is read. This is the dew point.

But how does this enable us to find the relative humidity? Suppose the temperature of the air was 15° C. and that the dew point was 5° C. While the air was being cooled down from 15° to 5°, the pressure of the aqueous vapour in it did not alter (though of course the capacity of the air for holding that vapour was all the time becoming less). But at 5° the air was saturated with vapour.

∴ the pressure was .65 cms. (this being the maximum pressure of aqueous vapour at 5° C.)

$\therefore$ when the air was at 15° C., the pressure of the vapour contained in it was also .65 cms.

Now the mass of vapour in a given volume of air will be in proportion to the pressure of that vapour.

$$\therefore \text{Rel. humidity} = \frac{\text{mass of vapour actually present}}{\text{mass of vapour required to saturate}}$$

$$= \frac{\text{pressure of vapour actually present}}{\text{max. pressure of vapour at 15° C.}}$$

$$= \frac{\text{max. pressure of vapour at 5° C.}}{\text{max. pressure of vapour at 15° C.}}$$

$$= \frac{.65}{1.27}$$

$$= .51 \text{ or } 51\ \%.$$

It will be seen from the above that, in general,

$$\textit{rel. humidity} = \frac{\textit{max. pressure of vapour at dew point}}{\textit{max. pressure at existing temperature}}$$

52. Formation of Dew. In the "shaving tin" experiment a deposit of dew was produced by cooling the air artificially. A similar result is often produced in Nature, for as evening comes on, the various objects on and near the earth (grass, stones, etc.) rapidly lose their heat by radiation (§92). As a result the temperature of these objects, and therefore of the air in contact with them, falls to the dew point, and dew is deposited on them.

Sometimes the cooling effect is so great that the temperature of large masses of air near the ground is reduced below the dew point. The vapour then condenses round the numerous particles of dust suspended in the atmosphere, and we have a *mist*. If the effect is very strongly marked it is known as a *fog*.

This condensation may take place at some distance above the earth's surface, and in that case *clouds* are produced.

In winter *hoar frost* is often observed. This happens when the dew point is below 0° C., the water vapour in that case being precipitated directly in the solid form.

[Q.—Why does an early morning mist clear away after the sun has been shining for a while ?]

53. Hygrometers. A number of instruments are in use for determining the relative humidity of the air, and most of them involve the determination of the dew point by the cooling of a polished surface, as described above.

The *wet and dry bulb* hygrometer, however, works in a different way. It consists of a pair of thermometers, one of which is quite ordinary, while the other has wrapped round its bulb a piece of muslin dipping into a small cup of water. The muslin sucks up the water, and so is always moist. This moisture evaporates, the necessary latent heat being drawn from the bulb of the thermometer. Thus the wet bulb thermometer shows a lower reading than the dry bulb instrument.

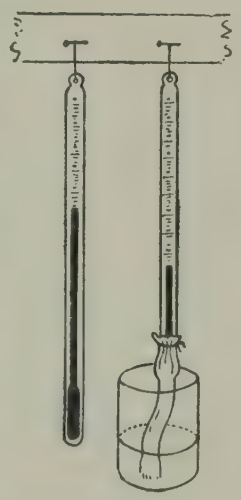

Fig. 32.
Wet and Dry Bulb Hygrometer.

Now the difference between the two readings is closely dependent on the cooling effect, which in turn depends on the rate of evaporation, which, again, depends on the relative humidity of the air. Hence there is a connection between the thermometer readings and the relative humidity. The connection is not a simple one, but it has been worked out, and for any given readings of the two thermometers the relative humidity can be found at once by reference to tables.

QUESTIONS ON CHAPTER III.

1. Heat is supplied at a uniform rate to a block of ice until it is converted into steam. Give a *brief* statement of the changes in volume condition and temperature which take place, and of the relative times occupied in the various stages of the process. —*Lond.* 1923.

2. A block of ice at —10° C. is put into a room which is kept at 20° C. How will the temperature and volume of the ice change as time goes on?

Draw approximate graphs, showing, (a) temperature and time, (b) volume and temperature.—*Oxf.* 1924.

3. Show by means of rough graphs how the volume of a given mass of water substance varies when it is heated at atmospheric pressure from —5° C. to 105° C. Explain with the help of your diagram why water would be quite unsuitable for filling thermometers, and point out the special importance in the economy of Nature of the peculiarities shewn by the curve.—*Camb.* 1923.

4. Give evidence that heat is evolved in the processes of condensation and solidification.—*Lond.* 1923.

5. You are asked to find by experiment the cooling curves of water and paraffin wax between the temperatures of 70° C. and 40° C., and to draw these curves on the same diagram. Describe your experiment, laying stress on the precautions you would take.

Draw these curves (approximately), say what you can deduce from them, and explain the differences between them. [Melting point of paraffin wax = 54° C.]—*Scot.L.C.* 1927.

6. Describe an experiment to illustrate the phenomenon of regelation.

Why is it difficult to make a snowball from snow which is much below its melting point?—*Camb.* 1925.

7. Distinguish between boiling and evaporation. Describe carefully how you would determine, (a) the boiling point of a solution, (b) the boiling point of the solvent.—*Lond.* 1922.

8. Distinguish between evaporation and ebullition. What factors affect the boiling point of a liquid, and how would you investigate the effect of one of them?—*Camb.* 1925.

9. Describe an apparatus suitable for making an accurate determination of the boiling point of a liquid.

How would the observations which you would make with such an apparatus be affected by impurities in the liquid?—*Oxf.* 1925.

10. Point out clearly the differences between boiling and evaporation. Describe and explain an experiment in which water is made to boil below 100° C.—*C.W.B.* 1924.

11. Distinguish between evaporation and boiling. Explain why wet clothes are likely to cause a chill, and why a boiler whose exit pipe is choked may explode.—*Camb.* 1923.

12. Describe an experiment to illustrate the term, *the maximum vapour pressure of a liquid.*

A thin walled glass bulb, hermetically sealed, contains 5 c.c. of water. It is placed in a 500 c.c. flask containing dry air at a pressure of 755 mm. of mercury, and the flask is securely corked.

What will be the pressure in the flask if the vessel containing the water is broken? The maximum vapour pressure of water at the temperature of the experiment is 15 mm.—*Camb.* 1924.

13. Explain the full significance of the expression, *maximum vapour pressure of a liquid at a given temperature.*

How would you measure this in the case of ether, at the ordinary temperature of the laboratory.—*M.L.L.S.B.* 1926.

14. By what experiments would you find out whether the air in the laboratory is saturated with water vapour or not?

In what ways does air saturated with water vapour differ from fog or mist?—*Oxf.* 1924.

15. Distinguish between saturated and unsaturated vapours. Describe a simple experiment whereby the saturation pressure of water vapour at different temperatures may be determined. *C.W.B.* 1922.

16. Describe the behaviour of a liquid which is allowed to evaporate, (a) in the open air, (b) in a closed space.

A barometer tube one metre long is filled with mercury and inverted in a dish of mercury, and a volatile liquid (ether) is allowed to enter the tube one drop at a time. Describe and explain carefully what you would observe.—*Camb.* 1924.

17. How does the temperature at which water boils depend upon pressure? Describe how you would verify your answer experimentally.—*Lond.* 1924.

18. Describe a method of determining the dew-point, and explain how you would find the pressure of aqueous vapour in the air and the relative humidity.—*Camb.* 1924.

19. What do you mean by a *saturated vapour*?

What test would you apply to a vapour to find out whether it is saturated or unsaturated?

Describe an experiment by which you could determine the vapour pressure of water at 100° C., and state what result you would expect to obtain.—*Oxf.* 1924.

20. In weaving cotton successfully it is necessary that the relative humidity of the air in the factory should not drop below a certain point. Describe any one way in which the relative humidity can be ascertained, and state how it could be raised in a factory when required.—*Oxf.* 1923.

Chapter IV.

THE MEASUREMENT OF HEAT.

54. Heat and Temperature. The student working through a revision course will hardly need to be reminded that heat and temperature are very different things. The measurement of temperature is carried out by means of a thermometer ; the measurement of heat is a more complicated process.

55. Units. The unit of temperature is the *degree* (Centigrade or Fahrenheit). The unit of heat is the ***calorie,*** which is defined as **the quantity of heat required to raise the temperature of 1 gram of water through 1° C.**

This definition implies that the same amount of heat is required to raise the temperature of 1 gm. of water from say 1° to 2°, or 30° to 31°, or 75° to 76°. This is not quite true, though it is so near the truth that we take it for granted in ordinary calculations. If great exactness is required, we define the calorie as "*the quantity of heat required to raise the temperature of* 1 *gram of water from* 4° *C. to* 5° *C.*"

For many purposes such a unit is inconveniently small. Thus gas companies employ the ***therm,*** which is equal to ***100,000 British thermal units*** (B.Th.U.), the latter being **the amount of heat required to raise the temperature of 1 lb. of water 1° F.**

Example 14. How many calories would be equal to 1 British thermal unit ? [1 pound = 454 grams.]

1 B.Th.U. = heat required to raise temp. of 1 lb. of water 1° F.
= heat required to raise temp. of 454 gms. of water $\frac{5}{9}$° C.
= $454 \times \frac{5}{9}$ calories = **252 calories.**

Example 15. If the combustion of 1 cubic foot of coal gas gives 560 B.Th.U., how many cubic feet of gas must be burnt

to raise 10 lbs. of water from freezing point to boiling point, assuming that 30 % of heat goes up the chimney ?—*Oxf.* 1924.

The difference between freezing point (32° F.) and boiling point (212° F.) is 180° F.

To raise temp. of 1 lb. of water 1° F. requires 1 B.Th.U.

∴ to raise temp. of 10 lbs. of water 180° F. requires $10 \times 180 = 1{,}800$ B.Th.U.

But, owing to waste, only 70 % of the heat produced by the gas is actually used in heating the water,

∴ heat produced by gas must be $100/70 \times 1{,}800$ B.Th.U.

But 1 cubic foot of gas gives 560 B.Th.U.

∴ number of cubic feet $= (\frac{100}{70} \times 1{,}800) \div 560 =$ **4.59.**

56. Thermal Capacity. The *thermal capacity or capacity for heat of a body* is the number of calories required to raise the temperature of that body by 1°C.

57. Water Equivalent. In experiments connected with the measurement of heat ("calorimetry") we often employ a sort of metal beaker known as a *calorimeter*, and it is usually necessary to know the *water equivalent* of this. **The *water equivalent of a calorimeter* is the number of grams of water which will be raised 1° C. by the heat required to raise the temperature of the calorimeter 1° C.**

Suppose the water equivalent of a calorimeter = w gms. Then thermal capacity of calorimeter

= no. of calories required to raise temp. of calorimeter 1° C.

= no. of calories required to raise w gms. of water 1° C.

= w (calories).

Thus thermal capacity of calorimeter is *numerically* equal to its water equivalent (only *numerically*, because thermal capacity is expressed in calories, while water equivalent is expressed in grams.)

58. Determination of Water Equivalent. A calorimeter is weighed—first empty, and then about a third full of water. The temperature is taken, and then a quantity of hot water of known temperature is added. After mixing, the temperature is again observed. Finally

the whole is weighed in order to find how much hot water has been added. Consider the following example :—

59. Example 16. In determining the water equivalent of a calorimeter, the following results were obtained :—
Weight of calorimeter, 56.8 gms.
Calorimeter + water, 83.3 gms.
Final weight (after adding hot water), 115.6 gms.
Initial temperature, 1.4° C.
Temperature of added water, 68.6° C.
Final temperature, 33.0° C.
Calculate the water equivalent.

Let required W.E. $= w$ gms.
Weight of water at 1.4° C. $= 85.3 - 56.8 = 28.5$ gms.
Weight of hot water added $= 115.6 - 85.3 = 30.3$ gms.
Heat given out by 30.3 gms. of hot water, cooling from 68.6° to 33.0° $= 30.3\ (68.6 - 33.0) = 30.3 \times 35.6$ cals.
Heat taken in by, (i.) 28.5 gms. of water, rising in temperature from 1.4° to 33.0° $= 28.5\ (33.0 - 1.4) = 28.5 \times 31.6$ cals.
(ii.) the calorimeter, equivalent to w gms. of water, also rising from 1.4° to 33.0° $= w\ (33.0 - 1.4) = 31.6\ w$ cals.
Total heat taken in $= (28.5 \times 31.6) + 31.6\ w.$
$= 31.6\ (28.5 + w).$
Now *heat given out* = *heat taken in*.
$\therefore\ 30.3 \times 35.6 = 31.6\ (28.5 + w).$
Solving the equation, we obtain, $w =$ **5.6 gms.**
(N.B.—The expression for "heat taken in" might have been obtained more shortly by regarding the total amount of water to be treated as $(28.5 + w)$ gms. Then heat required $= (28.5 + w) \times 31.6$ cals., as before.

60. Precautions against Error. In practically all calorimetric work there is one source of error against which precaution has to be taken, viz., the calorimeter and its contents tend to lose heat to the outside air (if they are above room temperature) or to gain heat from it (if they are below room temperature). Accordingly,

(i.) the calorimeter is placed in a larger containing vessel, and the space between the two is packed with felt, cotton wool, etc.

(ii.) the initial temperature is adjusted. Suppose, for instance, that in the "Water Equivalent" experiment just mentioned, a rough preliminary experiment shows that we may expect a rise of temperature of about 24°,

and that room temperature is 16°. We adjust the initial temperature to 4°, so that we start as much *below* room temperature as we shall finish *above* it. Thus during the first part of the experiment there is a little gain of heat from the room, and during the second part there is an equal loss.

61. Specific Heat. Suppose we took three quite similar calorimeters, each containing say 50 gms. of water at room temperature, and into the first we dropped 20 gms. of copper at 100°, into the second 20 gms. of mercury at 100°, and into the third 20 gms. of water at 100°. We should find, after stirring, that the final temperatures were very different, the hot water having giving out much more heat than the copper, and the copper than the mercury.

And just as water in the process of cooling gives out more heat than an equal mass of copper, and copper than mercury, so equal masses of these (and other) different substances require different amounts of heat to raise their temperatures by equal amounts—they have different *specific heats*.

The *specific heat* of a substance is the ratio $x : y$, where y is the quantity of heat required to raise the temperature of a certain mass of water through 1° C., and x is the quantity of heat required to raise the temperature of an equal mass of the given substance through 1° C.

Suppose the temperature of m gms. of a substance is raised 1° C., and similarly the temperature of m gms. of water, and let the specific heat of the substance be s.

Now to raise the temperature of 1 gm. of water by 1° C. requires 1 cal.

$\therefore$ to raise the temperature of m gms. of water by 1° C. requires m cals.

$\therefore$ to raise the temperature of m gms. of the substance by 1° C. requires ms cals. (by definition of specific heat).

$\therefore$ to raise the temperature of 1 gm. of the substance by 1° C. requires s calories.

Hence *the specific heat of a substance is numerically equal to the number of calories required to raise the temperature of* 1 *gm. of it by* 1° *C.*, a result of which we shall frequently make use.

Example 17. The capacity for heat of a certain volume of a given liquid is one-third that of an equal volume of water. The density of the liquid is 0.65 gms. per c.c. Calculate its specific heat.—*O.C.* 1924.

Consider 1 c.c. of water, and of the given liquid.
The capacity for heat of 1 c.c. of water (= 1 gm.) is 1 calorie.
∴ capacity for heat of 1 c.c. of the liquid (= 0.65 gms.) is $\frac{1}{3}$ cal.
i.e., to raise temp. of 0.65 gm. 1° C. requires $\frac{1}{3}$ cal.
∴ to raise temp. of 1 gm. 1° C. requires $\frac{1}{3} \div .65 = .513$ cals.
i.e., **specific heat = .513.**

Example 18. Compare the thermal capacity of mercury with that of an equal volume of alcohol (specific heat of mercury = .033 and of alcohol = .61 ; relative density of mercury = 13.6 and of alcohol = .79).

Consider v c.c. of each.
Mass of v c.c. of mercury = $13.6\ v$ gms.
Heat required to raise temperature 1° C. (i.e., thermal capacity) $= 13.6\ v \times .033 = .449$ cals.
Similarly, thermal capacity of v c.c. of alcohol $= .79\ v \times .61 = .482$ cals.

$$\therefore \frac{\text{thermal capacity of mercury}}{\text{thermal capacity of alcohol}} = \frac{.449}{.482} = \frac{1}{1.07}$$

62. Determination of Specific Heat. Suppose we are finding the specific heat of lead shot for instance. A calorimeter is weighed, first empty and then about half full of water. A quantity of lead shot is heated to a known temperature, and is then plunged into the water, the temperature of which is taken before the lead enters it and then afterwards. The weight of lead must be known, and the result is then calculated as in the following example :—

Example 19.
Weight of water, 48.5 gms.
Weight of lead, 82.1 gms.
Initial temperature of lead, 98.4° C.
Initial temperature of water, 15.6° C.
Final temperature of mixture, 19.3° C.

Let s = required specific heat.

1 gm. of lead, cooling 1° C., would give out s cals.

$\therefore$ 82.1 gms. of lead, cooling from 98.4 to 19.3, give out $82.1 \times (98.4 - 19.3) \times s$ cals.

Heat taken in by 48.5 gms. of water rising from 15.6° to 19.3° $= 48.5\ (19.3 - 15.6) = 48.5 \times 3.7$ cals.

But *heat given out by lead = heat taken in by water.*

$\therefore\ 82.1 \times s\ (98.4 - 19.3) = 48.5 \times 3.7.$

$$s = \frac{48.5 \times 3.7}{82.1 \times 79.1} = \mathbf{.028.}$$

In the above calculation we have not allowed for the W.E. of the calorimeter; for not only does the 48.5 gms. of water rise in temperature, but also the calorimeter in which the water is contained.

Suppose the calorimeter is made of copper (S.H., .095) and weighs 59 gms.; then its W.E. $= 59 \times .095 = 5.6$ gms. (or we could have found the W.E. by the experiment described in §59.)

"*Heat taken in*" now becomes $(48.5 + 5.6) \times 3.7$, and our equation will be $82.1 \times s\ (98.4 - 19.3) = (48.5 + 5.6) \times 3.7.$

$$\therefore\ s = \frac{54.1 \times 3.7}{82.1 \times 79.1} = \mathbf{.031}$$

63. The method just described is known as the **Method of Mixtures.** It may be used for finding the specific heat of a solid, or of a liquid which can be added to water without the evolution of heat. (It could *not* be used, for instance, to find the specific heat of alcohol, which becomes warm when water is added to it.)

In cases such as that just mentioned a modification of the method is employed, a solid of known specific heat being used. Consider the following example:—

Example 20. 25 gms. of iron nails (S.H. = .11) at 96.4° C., are mixed with 30 gms. of alcohol at 16.4° C., in a calorimeter of which the W.E. is 6.2 gms. The final temperature is 24.4° C. Find the specific heat of the alcohol.

Let s = required specific heat.

Heat *given out* by 25 gms. of iron, S.H. = .11, cooling from 96.4° to $24.4° = 25 \times .11 \times (96.4 - 24.4) = 25 \times .11 \times 72$ cals.

Heat *taken in*, (i.) by alcohol $= 30 \times s \times (24.4 - 16.4) = 30 \times s \times 8.$
(ii.) by calorimeter (W.E. = 6.2 gms.) $= 6.2 \times (24.4 - 16.4) = 6.2 \times 8.$

$\therefore\ 25 \times .11 \times 72 = (30 \times s \times 8) + (6.2 \times 8).$

$\therefore\ 198 = 240\ s + 49.6.$

$$\therefore\ s = \frac{198 - 49.6}{240} = \mathbf{.62}\ \text{nearly.}$$

64. The Method of Mixtures was at one time used occasionally for the measurement of high temperatures, e.g., that of a furnace. The method will be sufficiently indicated by the following example :—

Example 21. A mass of copper, weighing 50 gms., was placed in a furnace and then rapidly transferred to 250 gms. of water at 10° C., contained in a vessel whose water equivalent is 15 gms. The temperature of the water rises to 21.3° C. Find the temperature of the furnace (specific heat of copper = .096).—*Camb.* 1921.

Let required temperature = t° C.
Heat lost by copper = $50 \times .096\ (t - 21.3)$.
Heat gained by water and calorimeter = $(250 + 15)\ (21.3 - 10)$
$= 265 \times 11.3$.
$\therefore\ 50 \times .096\ (t - 21.3) = 265 \times 11.3$.
Solving the equation, we get, $t =$ **645° C.**

65. Latent Heat of Fusion of Ice. The *latent heat of fusion of ice* is the number of calories required to convert 1 gram of ice at 0° C. into water at the same temperature.

In order to find it, some pieces of ice at the melting point are dried on blotting paper, and quickly introduced into a weighed quantity of water in a calorimeter, of which the temperature has been taken. After stirring until all the ice is melted, the temperature is again observed. Finally, the calorimeter is again weighed, the increase in weight giving the amount of ice introduced. The result is then worked out as shown in the following example :—

Example 22. A copper calorimeter, weighing 58.2 gms., contains some water, the whole weighing 128.7 gms. A quantity of dry ice, at 0° C., is then introduced, and this causes the temperature to fall from 19.6° C. to 7.7° C. The final weight is 139.0 gms. Find the latent heat of ice, taking into account the water equivalent of the calorimeter. (Specific heat of copper = .095.)

Let l = required latent heat.
W.E. of calorimeter = $58.2 \times .095 = 5.5$ gms.
Weight of water = $128.7 - 58.2 = 70.5$ gms.
Weight of ice = $139.0 - 128.7 = 10.3$ gms.
Heat given out by water and calorimeter, cooling from 19.6° to 7.7° = $(70.5 + 5.5)\ (19.6 - 7.7) = 76 \times 11.9$ cals.

Heat taken in (i.) to convert 10.3 gms. of ice at 0° C. into water at 0° C. = 10.3 l cals.

(ii.) to raise the water thus formed from 0° C. to 7.7° C. = 10.3×7.7 cals.

$\therefore 76 \times 11.9 = 10.3\ l + (10.3 \times 7.7)$.

$\therefore 904.4 = 10.3\ l + 79.3$.

$\therefore 10.3\ l = 825.1$.

$\therefore l =$ **80.1**

66. Somewhat similar principles are involved in the following calculation, which suggests a means of finding the specific heat of ice :—

Example 23. 10 gms. of ice at —10° C., were mixed with 100 gms. of water at 20° C., contained in a vessel whose thermal capacity (or water equivalent) was 5. The final temperature of the mixture was 10.8° C. If the latent heat of water = 80, calculate the specific heat of ice.—*Oxf.* 1925.

Let s = required specific heat.

Heat taken in, (i.) by 10 gms. of ice, rising in temperature from —10° C. to its melting point, 0° C. $= 10 \times 10 \times s = 100\ s$ calories.

(ii.) by 10 gms. of ice in melting $= 10 \times 80 = 800$ cals.

(iii.) by 10 gms. of *melted* ice, rising in temperature from 0° to the final temperature of 10.8° C. $= 10 \times 10.8 = 108$ cals.

$\therefore$ total number of cals. taken in $= 100\ s + 800 + 108$.

Heat given out by 100 gms. of water and calorimeter (= 5 gms. water) cooling from 20° to 10.8° $= (100 + 5)\ (20 - 10.8) = 105 \times 9.2 = 966$ cals.

But *heat taken in = heat given out.*

$\therefore 100\ s + 800 + 108 = 966$.

Solving this equation, we obtain $s =$ **0.58.**

67. Latent Heat of Vaporisation. The *latent heat of vaporisation of water* is the number of calories required to convert 1 gram of water at the boiling point into steam at the same temperature. The boiling point of water depends on the pressure (§47), but in this book we shall deal only with the latent heat of vaporisation at 100° C.

68. Determination of Latent Heat of Vaporisation Water is boiled in a flask, A, and the steam passes through a glass tube, B, sloping upwards and packed round with felt, into some water contained in a calorimeter, C. The calorimeter is weighed, (i.) before and (ii.) after passing the steam, and the increase gives

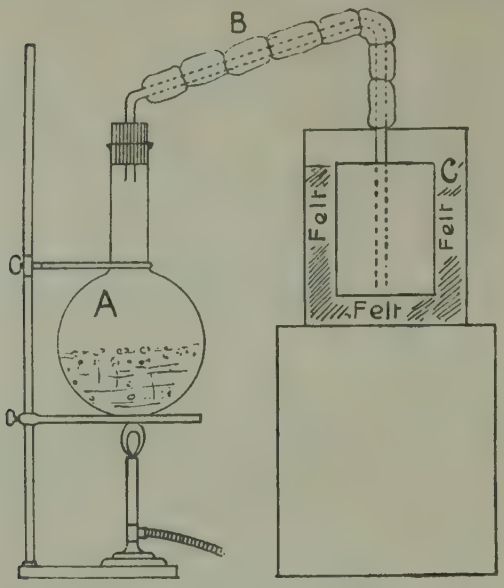

Fig. 33. Latent Heat of Steam.

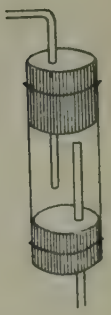

Fig. 33a.
Steam Trap.

the weight of steam condensed. The temperature is also noted before and after the steam has been passed, and the result is worked out as follows :—

Example 24. In an experiment to find the latent heat of steam, steam was passed into a quantity of water at 5.3° C., and the temperature rose to 27.4° C. The calorimeter weighed 39.42 gms., and the calorimeter with water (before passing the steam) weighed 87.30 gms. After passing the steam, the total weight was 89.09 gms. Find the latent heat, neglecting the water equivalent of the calorimeter.

Let L = required latent heat.

Weight of water in calorimeter = 87.30—39.42 = 47.88 gms.

Weight of condensed steam = 89.09—87.30 = 1.79 gms.

Now as L calories are required to convert 1 gm. of water at 100° C. into steam at 100° C. (defn.), it follows that 1 gm. of steam at 100° C. will *give out* L cals. as it condenses to water at 100°,

$\therefore$ heat given out by condensing of 1.79 gms. of steam into water at 100° = 1.79 L cals.

As this water at 100° cools to the final temperature of 27.4°, it will give out a further 1.79 (100—27.4) cals.

Total heat given out = 1.79 L + 1.79 (100—27.4) cals.

Heat taken in by 47.88 gms. of water rising in temperature from 5.3° to 27.4° = 47.88 (27.4—5.3).

$\therefore$ 1·79 L + 1.79 (100—27.4) = 47.88 × 22.1.

Solving this, we obtain L = **519 cals.**

Precautions to secure a good result. See §60 for two precautions common to practically all calorimetric measurements.

In the particular case we are considering, however, certain additional precautions are taken to reduce the error caused by the steam condensing before it enters the water in the calorimeter. This would evidently cause the rise in temperature to be less than it should be. These precautions are :—

(iii.) B is surrounded by felt.

(iv.) It slopes *upwards* so that condensed steam may run back into the flask.

(v.) The end part of the delivery tube, which turns vertically downwards, is made as *short* as possible.

(vi.) Steam is allowed to pass through the tube for a minute or two before putting the tube into the calorimeter.

(vii.) A steam-trap (fig. 33a) is often inserted in the path of the tube, just before it enters the calorimeter. This is of somewhat doubtful advantage because it tends to nullify the precaution taken in (v.)

Lastly, after the steam has been passed, the temperature of the water in the calorimeter is often rather high, and therefore

(viii.) No time must be lost in making the final weighing, otherwise there is considerable loss by evaporation.

69. Specific Heat by Method of Cooling. The principle underlying this method is known as "**Newton's Law of Cooling,**" and may be stated thus : **The rate at which a hot body loses heat is proportional to the difference between its own temperature and that of its enclosure, and does not depend on any other circumstance except the nature and extent of its surface.** For our present purpose, the special point to notice is that the rate at which a body loses heat does *not* depend upon its specific heat.

A calorimeter is about two-thirds filled with hot water, and a thermometer is supported in it as indicated in

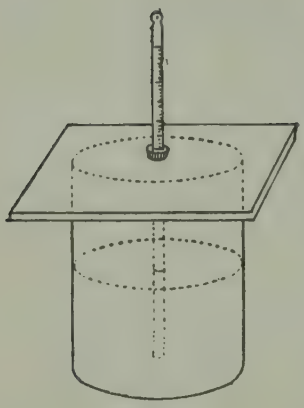

Fig 34.

Specific Heat by Method of Cooling.

fig. 34. The time taken for the liquid to cool through a given range of temperature (say 50° to 45° C.) is carefully noted. An equal *volume* of the liquid under examination, say alcohol, is now substituted for the water, and the time of cooling for the same range of temperature is again noted. The result is then calculated as in the following example :—

Example 25. A calorimeter, the water equivalent of which is 10 gms., is found to cool from 60°C. to 50° C. in 8 minutes when containing 90 gms. of water, and in 4 minutes when containing 60 gms. of another liquid. Calculate the specific heat of the liquid, and give practical directions for performing the experiment.—*Camb.* 1921.

Let s = specific heat of the liquid.

When the water cooled from 60° C. to 50° C., the loss of heat by the water was $90 \times 10 = 900$ cals., and by the calorimeter was $10 \times 10 = 100$ cals.

$\therefore$ total loss of heat was 1,000 cals.

This loss took place in 8 minutes.

$\therefore$ rate of loss was $1{,}000 \div 8 = 125$ cals. per minute.

Now consider the other liquid. Its loss of heat in cooling from 60° C. to 50° C. was $60 \times 10 \times s = 600s$ calories, and the heat lost by the calorimeter was $10 \times 10 = 100$ cals.

$\therefore$ total loss was $(600s + 100)$.

$\therefore$ rate of loss was $(600s + 100) \div 4 = 150s + 25$ cals per min.

But rate of loss is the same in each case (Law of Cooling).

$$\therefore 125 = 150s + 25.$$

Solving this, we obtain $s = .67$.

N.B.—To verify the Law of Cooling (which is, however, only approximately true) we should evidently proceed in the reverse manner, i.e., work with liquids of known specific heat.

70. Heat and Energy. So far we have measured heat in terms of the calorie. We shall see presently how it may be measured in *mechanical* units. It will be advisable first of all, however, to revise our ideas of what such terms as force, energy, etc., really mean.

***Force* is that which changes, or tends to change, a body's state of rest or motion.**

Thus *friction* is a force, because it tends to stop one body from moving over another one. One of the commonest forces, however, is the *weight* of a body—the force with which the earth pulls it. Thus if an object is lying near the edge of a table, and we give it a slight push so that it falls over the edge, it is its weight which "changes the body's state of motion"—from horizontal motion to nearly vertical.

One unit of force is the *weight of one pound*, and we must take care to distinguish between the stuff itself, the *mass*, as it is called, and the force with which the earth pulls that mass. All sorts of forces could be measured in terms of the pound weight as unit. We could, for instance, measure the pull of an engine on a train, by inserting a spring balance (it would certainly have to be a strong one!) between engine and train, and taking the reading.

Work. *A force is said to do work when its point of application moves in the direction in which the force acts.* A man pulling at a rope attached to a block of granite, may be exerting great force (he "tends to change a body's state of rest") but he does no *work* unless he makes the block move.

A convenient unit of work is the ***foot-pound*,** which is **the work done when a force of one pound moves its point of application through a distance of one foot.** If a man lifts a 7 lb. weight through a vertical height of 3 feet, the work done is equal to 7×3 or 21 foot-pounds.

71. Energy. *Energy means capacity for doing work.* The 7 lb. weight just considered might be made to do work as it descends 3 feet to its original position—it might, for instance, by suitable attachment, be made to drive the works of a clock, and the energy it possesses would be equal to 21 foot-pounds.

A moving body always possesses energy, i.e., it is capable of doing work. Air in motion is capable of driving a windmill; a bullet in motion is capable of

knocking down a target ; and so on. The energy possessed by a moving body is known as *kinetic* energy, every other form of energy (e.g., that possessed by a coiled-up spring) being known as *potential* energy.

It is shown in Mechanics that if a body of mass m lbs. is moving with a velocity of u feet per second, then its kinetic energy is equal to $\frac{1}{2}\ mu^2/g$ foot-pounds, where g is the acceleration due to gravity ($=32$ feet per sec. per sec.)

Thus if a body of mass 20 lbs., is moving with a velocity of 8 feet per second, its k.e. is equal to $\frac{1}{2} \times 20 \times 8^2/32 = 20$ foot-pounds.

It will be convenient here to define the terms *Power* and *Horse-power*.

***Power* means the rate of doing work,** and ***a Horse-power* is a rate of working of 550 foot-pounds per second.**

72. How our ideas on Heat have developed.

(1.) Until 1798, heat was believed to be an elastic fluid, without weight, which filled up the tiny spaces between the particles of all kinds of matter. This fluid was called *caloric*. If a substance was struck hard for a number of times (e.g., a piece of iron by a hammer) some of the caloric was squeezed out and might flow into a thermometer or into one's hand. Hence it "felt hot."

A body also became hot by friction. This, it was thought, was because a little of the substance was rubbed off as a powder, and the powder had a less capacity for holding the caloric than had the original block of substance. Hence the excess of caloric escaped.

(2.) In 1798 Count Rumford arranged an experiment in which a blunt borer was rotated for a long time inside a hole in a mass of gun-metal. Very little metal was rubbed off, and yet much heat was produced. Rumford argued that as there was a *large* quantity of heat, a *large* quantity of motion (rotation of borer) and only a *small* quantity of borings, the heat was much more likely to have proceeded from the motion than from the borings. Hence he concluded that "heat is a mode of motion."

(3.) In 1799 Davy, by rubbing together two pieces of ice in a vacuum, succeeded in producing enough heat to melt the ice. Now those who held to the caloric theory had always maintained that water must contain more caloric than ice (because heat "added to" ice produces water.) Hence by squeezing out some of the caloric contained in ice, a substance containing *more* caloric is produced. The old theory thus led to an absurdity. Davy concluded that "friction does not diminish the capacity of bodies for heat."

(4.) In 1843 Dr. Joule finally settled the question by an experiment in which he showed that a given amount of energy always produced a fixed amount of heat. Thus he arrived at the modern view that *heat is a form of energy.*

Joule's method was to arrange two weights in such a way that in falling they operated a simple mechanism which caused a paddle to revolve with great friction through a quantity of water. The work done by each fall of a weight was easily found by multiplying the weight in pounds by the depth of fall in feet, this giving the work done in foot-pounds. Obviously the total work done is found by multiplying this answer by the total number of falls.

The heat produced could be measured in calories by multiplying the number of grams of water by the rise of temperature expressed in degrees Centigrade. (Joule actually used pounds and degrees Fahrenheit for his units of mass and temperature respectively.)

As a result of his experiments he showed that **778 foot-pounds of work are required to raise 1 lb. of water 1° F.** 778 foot-pounds would be said to be the *mechanical equivalent* of this particular unit of heat.

The following relationships should be carefully remembered :—

1 B.Th.U. = 778 foot-pounds.

1 calorie = 4.26×10^4 gram-centimetres.

(A gram-centimetre is the work required to raise a mass of 1 gram through a vertical height of 1 cm. ; cf. foot-pound.)

73. Mechanical Equivalent of Heat. Laboratory Method. Joule's appa.atus is much too complicated for the school laboratory. A rough value for the mechanical equivalent can be found, however, by placing about a pound of lead shot in a long cardboard tube closed at each end by a cork. The temperature of the shot is taken by means of a sensitive thermometer, and the tube is then repeatedly inverted, at least fifty times, causing the shot to fall again and again from one end of the tube to the other. The final temperature is then observed, and the depth through which the shot has been falling is measured. Thus suppose :—

Mass of lead shot = 600 gms.
Fall = 85 cms.
No. of inversions = 60.
Rise in temp. = 17.3—13.4 = 3.9° C.
The specific heat of lead is supposed to be known = .031.
Then work done in 1 inversion = 600×85 gm.-cm.
$\therefore$ work done in 60 inversions = $600 \times 85 \times 60$ gm.-cm.
Heat produced = $600 \times .031 \times 3.9$ calories.

$$\therefore \text{ work equivalent to 1 calorie} = \frac{600 \times 85 \times 60}{600 \times .031 \times 3.9}$$
$$= 4.2 \times 10^4 \text{ gm.-cm.}$$

74. Experiments and Observations to show that Heat is a Form of Energy. Some of these will have already been suggested by what has been said of Rumford, Davy and Joule in §72. A few others are added here :—

(a) When a boy rubs a metal button to and fro on his coat sleeve, it becomes hot.

(b) A bicycle pump soon becomes hot when in use.

(c) A drill becomes hot when used for boring wood or metal.

In the above cases kinetic energy is transformed into heat.

(d) Heat from burning coal in a steam engine is transformed into the kinetic energy represented by the moving train.

(e) Heat from the sun is constantly turning large quantities of water into vapour. It is therefore lifting the water against the force of gravity—"doing work against gravity."

These and many other examples show that energy can be transformed into heat, and vice versa.

75. Efficiency of a Steam Engine. In Joule's experiment a quantity of mechanical energy was practically *completely* converted into heat. But we cannot *completely* turn a quantity of heat into mechanical energy. In practice not more than 17 % of the heat of combustion is transformed into mechanical work even by the best condensing engines of the quadruple expansion type, i.e., the *efficiency* of such an engine is 17 %. The efficiency of an ordinary locomotive is not more than 8 %.

These low efficiencies are due chiefly to the fact that much of the heat supplied to the engine escapes with the exhaust steam into the condenser, or out into the atmosphere. Such loss could only be prevented by working with a condenser at absolute zero.

76. Sun's Heat as a Source of Energy. It is easy to show that all our ordinary sources of energy are derived from the heat of the sun; thus:—

(i.) *Coal.* This is the fossil remains of plants, which depended for their growth on their power to make starch from carbon di-oxide (contained in the air) and water. This process required energy, the source of which is sunlight. In the absence of sunlight, starch formation cannot take place.

(ii.) *Gas* and *Coke.* Obtained from coal.

(iii.) *Water.* (Waterfalls, a common source of energy.) For the part played by the sun, see §74.

(iv.) *Electricity.* The dynamos which produce this are driven by steam (coal required) or by water power.

(v.) *Wind.* Really convection currents in the earth's atmosphere, and these are set up by the heat of the sun.

77. Methods of Producing Heat. These all depend on the fact that heat is a form of energy, and can only be obtained from some other form, e.g.,

(i.) *Chemical Energy.* We turn this into heat when we burn coal, gas and other combustible substances.

(ii.) *Electrical energy.* Used as a source of heat in the electric oven, electric iron, etc.

(iii.) *Mechanical Energy.* Often "accidentally" turned into heat (working of bicycle pump, brake drum of motor car, etc.) We make use of it in striking a match, and the South Sea Islander uses it in obtaining fire.

78. We will conclude this chapter with the working of two typical examples:—

Example 26. Explain what is meant by the efficiency of a heat engine.

Find the efficiency of a gas engine which consumes 59 cub. ft. of gas per horse-power hour, the calorific value of the gas being 152 British thermal units per cub. ft. Given that 1 British thermal unit is equivalent to 778 ft. lbs —*C.W.B.* 1923.

For meaning of "efficiency," see §75.

1 horse-power hour = work done in 1 hour by engine working at 1 horse-power.

1 horse-power = 550 ft. lbs. per sec.
= 550 × 3,600 ft. lbs. per hour.

59 cub. ft. of gas yield 59×152 B.Th.U. $= 59 \times 152 \times 778$ ft. lbs.

$$\therefore \text{efficiency} = \frac{\text{work done}}{\text{mechanical equivalent of heat supplied.}}$$

$$= \frac{550 \times 3{,}600}{59 \times 152 \times 778.}$$

$$= .2838 \text{ (or 28.38 per cent.)}$$

(N.B.—This efficiency is much higher than those mentioned in §75; but we are here dealing with an *internal combustion* engine.)

Example 27. A sleigh is pulled forward on a level road at 20 ft. per sec. over some melting snow by a constant force of 30 lbs.

Assuming that all the resistance is caused by friction with the snow, how much of this will be melted in 1 minute by the passing of the sleigh? The mechanical equivalent of heat is 780 ft. lbs. per British thermal unit, and the latent heat of fusion of ice is 144 B.Th.U. per lb.—*M.L.L.S.B.* 1926.

In 1 minute the sleigh advances $20 \times 60 = 1{,}200$ ft., against a constant force of 30 lbs.

$\therefore$ work done per minute $= 1200 \times 30$ ft. lbs.

Now 780 ft. lbs. $= 1$ B.Th U

$$\therefore 1200 \times 30 \text{ ft. lbs} = \frac{1200 \times 30}{780} \text{ B.Th.U.}$$

And 144 B.Th.U. melt 1 lb. of snow.

$$\therefore \frac{1200 \times 30}{780} \text{ B.Th.U. will melt } \frac{1200 \times 30}{780} \div 144.$$

$$= \textbf{0.32 lbs.} \text{ or } \textbf{5.1 ozs.}$$

QUESTIONS ON CHAPTER IV.

1. Explain the meaning of capacity for heat, specific heat and water equivalent. Describe fully how you would measure the specific heat of turpentine.

A calorimeter contains 300 grams of water, and is heated by a steady flame, the temperature rising 10° in 4 minutes. Under the same conditions, but with 400 grams of water, it requires 5 mins. for the same rise of temperature. Find the water equivalent of the calorimeter.—*C.W.B.* 1926.

2. Explain the meaning of the statement that the specific heat of paraffin oil is 0.52, and describe how you would determine this quantity experimentally, indicating the precautions necessary to obtain an accurate result.—*Lond.* 1921.

3. 100 grams of copper (specific heat .095) are heated to 100° C., and put into 150 grams of petroleum at 10° C., contained in a copper pot weighing 40 gms. The temperature rises to 19.5° C. Find the specific heat of petroleum.—*Lond.* 1924.

4. A bunsen burner is capable of raising the temperature of a kilogram of water from 0° C. to 40° C. in 5 minutes. How long would it take to raise the temperature of 400 gms. of oil from 10 C. to 230° C.?

The specific heat of the oil is 0.6.—*Camb.* 1923.

5. Explain thermal capacity.

A calorimeter contains 61.2 gms. of water at 15° C., and into it are poured 47.6 gms. of water at 40° C. The resulting temperature of the mixture is 25.4° C. What may be deduced from this experiment? Make the necessary calculations.

If the mass of the calorimeter is 44.5 gms., what further information may be obtained?—*C.W.B.* 1923.

6. (a) What is meant by the "thermal capacity' of an object?

(b) Into an empty metal calorimeter at 13.5° C. were poured 30 gms. of water at 32.2° C. After stirring, the temperature fell

to 31.5° C. The calorimeter weighed 11.7 gms. Calculate, (i.) the thermal capacity of the calorimeter, (ii.) the specific heat of the metal composing it.—*Oxf.* 1923.

7. A mass of copper weighing 50 gms. is placed in a furnace and then rapidly transferred to 250 gms. of water at 10° C., contained in a vessel whose water equivalent is 15 gms. The temperature of the water rises to 21.3° C. Find the temperature of the furnace. (Specific heat of copper = .096.)—*Camb.* 1921.

8. How is it possible to measure extreme temperatures, e.g., of a furnace or of liquid air?—*M.L.L.S.B.* 1922.

9. Explain the difference in the meanings of *temperature* and *quantity of heat.*

On dropping 100 gms. of steel at 315° C. upon 2,600 gms. of mercury at 15° C., the temperature of the liquid rose to 51° C. The specific heat of mercury is 033; what was that of the steel? —*M.L.L.S.B.* 1925.

10. Explain the meaning of specific heat.

If 1000 gms. of water at 45° C. are poured into 450 gms. of water at 15° C., standing in an aluminium vessel which weighed 250 gms., what will be the resulting temperature, the specific heat of the metal being 0.2?—*M.L.L.S.B.* 1926.

11. Describe how you would measure the latent heat of fusion of ice, mentioning any precautions you would take to increase the accuracy of the work.—*Lond.* 1921.

12. A substance has a melting point at 333° C, a specific heat when solid of 0.03 and a latent heat of fusion 5. Explain the meaning of these statements. Supposing heat to be supplied uniformly to a mass of this substance, compare the times of bringing it from 30° C. to its melting point and of melting it. —*Lond.* 1925.

13. A copper calorimeter of mass 50 gms. and specific heat 0.1, contains 183 gms. of water at 50° C. Ice is added till the temperature is reduced to 24° C., and the total mass of the calorimeter and its contents is now found to be 279 gms. What value does this give for the latent heat of fusion of ice? Discuss the probable sources of error in the experiment.—*Camb.* 1922.

14. Explain what is meant by the terms Specific Heat and Latent Heat.

100 gms. of ice at —10° C. are put into water at 0° C. Steam at 100° C. is passed into the mixture at the rate of 1 gm. each minute. Assuming that no heat is lost or gained by radiation during the process, find how long it will take for the ice to be just melted. (Specific heat of ice, 0.5. Latent heat of steam 540. Latent heat of water, 80.)—*M.L.L.S.B.* 1922.

15 Define clearly the terms, calorie, specific heat, latent heat; and describe one method of finding the specific heat of a liquid.

A copper calorimeter weighing 250 gms. contains 92.5 gms. of water at 15° C. 5 gms. of pounded ice at —4° C. are put into this water, and the mixture is stirred till the temperature is steady. Prove that, if there is no loss of heat by radiation, the resulting temperature should be about 11° C.

Specific heat of copper = 0.095. Specific heat of ice = 0.49. Latent heat of ice = 80.—*Scot. L.C.* 1926.

16. Define latent heat, specific heat.

A copper calorimeter (specific heat, 0.09) weighed 50 gms. empty, 156 gms. when containing a mixture of ice and water, and 168 gms. after steam had been passed in until the temperature was 20° C. What was the mass of ice originally in the calorimeter? (Latent heats of steam and water are 536 and 80 calories per gm. respectively.)—*Camb.* 1924.

17. Define latent heat, water equivalent. A calorimeter of water equivalent 12 gms., contains 25 gms. of ice, and steam is admitted to the calorimeter until 5 gms. have been condensed. Calculate the resulting temperature of the calorimeter and its contents. (Latent heat of fusion of ice = 80, latent heat of steam = 536.)—*Camb.* 1923.

18. Define the latent heat of steam at any temperature. Determine its numerical value from the following results:— 10 grams of dry steam at 100° C. was passed into 300 grams of water at 10° C., contained in a calorimeter of mass 50 grams, and specific heat 0.1. The final resulting temperature was 30° C.

Sketch the apparatus used in this experiment and point out what steps must be taken to obtain a satisfactory result. —*Lond.* 1927.

19. A mixture of 250 gms. of water and 50 gms. of ice is heated in an open vessel till it is all converted into steam at atmospheric pressure. State what you know concerning the changes in volume and temperature which occur. Determine the total amount of heat necessary, given that the latent heat of fusion of ice is 80 and the latent heat of steam 539 in Centigrade units.—*Lond.* 1926.

20 If, during a short stop at a station, you were given a cup of tea which was too hot to drink even if the milk available were added and if you wished to drink the tea before the train left the station, would you add the milk immediately or wait as long as possible before adding it? Give reasons for your answer.—*Camb.* 1925.

21. A calorimeter, the water equivalent of which is 10 gms. is found to cool from 60° C. to 50° C in 8 minutes when containing 90 gms of water, and in 4 minutes when containing 60 gms of

another liquid Calculate the specific heat of the liquid, and give practical directions for performing the experiment —*Camb.* 1921.

22. State the evidence which led Rumford to assert that heat was not a form of matter.

Describe any experiment which seems to you to be a conclusive proof that heat is a form of energy, pointing out clearly why it is that you consider the proof to be conclusive.—*Oxf.* 1925.

23. A tube 6 feet long containing a little mercury and closed at both ends is rapidly inverted fifty times. What is the maximum rise in temperature of the mercury that can be expected ?

(Specific heat of mercury = 1/30. 1 British Thermal Unit is equivalent to 778 foot-pounds.)—*Lond.* 1923.

24. Explain what is meant by the mechanical equivalent of heat and describe a method of measuring it.—*Camb.* 1924.

25. Define *unit quantity of heat* in British and Metric units respectively, and state the value of each in suitable mechanical units.

" Heat is a form of energy." Describe three examples, within your own observation, in illustration of this statement.—*Lond.* 1926.

26. Give a short account of the chief methods of production of heat—*Camb.* 1923.

27. Explain what meaning you attach to the phrase : " One pound-calorie of heat is equivalent to 1,400 foot-pounds of work."

What is the efficiency of an engine which consumes 50 lbs. of coal in an hour and works at 20 horse-power ?

(The combustion of 1 lb. of coal yields 8,000 pound-calories. 1 horse-power = 33,000 foot-pounds per min.)—*M.L.L.S.B.* 1921.

28. If all the heat of a furnace could be turned into useful work, what would be the horse-power of a steam engine which consumed 240 lbs. of coal per hour? 1 lb. of coal gives 12,500 British Thermal Units, each of which is equivalent to 770 ft. lbs. and 1 h.p. = 33,000 ft. lbs. per minute.)

In practice there is much waste of energy ; what are the chief causes of this ?—*M.L.L.S.B.* 1925.

29. If a lump of lead fell from a height of 100 ft. on a stone pavement, by how much would its temperature be raised, supposing that all the heat caused by the blow were retained by the metal ? A British Thermal Unit is equivalent to 780 ft. lbs., and the specific heat of lead is 0.03.—*M.L.L.S.B.* 1926.

30. What evidence is there for assuming that heat is a form of energy ? Write a short account of the methods by which the sun's energy is being utilised for the production of mechanical work.—*O.C.* 1925.

Chapter V.

HOW HEAT IS TRANSMITTED.

79. How a Hot Body Loses Heat. Suppose a hot body (e.g., a kettle containing hot water) is left for a while. How does it lose its heat ?

(i.) Some heat is *conducted* to the substance on which the kettle rests, and gradually makes its way through the substance.

(ii.) The air round the kettle becomes heated and consequently expands, thus becoming less dense than the surrounding air. The heated air consequently rises, and its place is taken by a fresh supply of cool air, the process being continually repeated. Thus the kettle loses heat to the air by *convection.*

(iii.) If we place our hands to one side of the kettle or a few inches below it, we feel the heat. This cannot be a case of convection, for by this process the air would travel *upwards* as just explained. The heat is actually reaching us by a process known as *radiation,* which will be described presently.

(In the case of an open vessel containing a hot liquid, *evaporation* is taking place, and the necessary latent heat is taken from the liquid ; hence there is an additional reason for loss of heat.)

80. Conduction. Suppose a bar of metal is divided into sections, A, B, C, etc., and suppose A is heated by means of a fire or bunsen burner. A " becomes hotter."

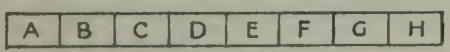

Fig. 35.

What is actually happening is that the vibrating molecules of which it is composed are made to vibrate with increased frequency. The neighbouring molecules of B consequently have *their* frequency increased. The molecules of A, however, have imparted some of their energy to the air molecules round about them, and therefore they cannot pass on the whole of their energy to the molecules of B.

The consequence is that the molecules of B vibrate somewhat less rapidly than the molecules of A, and so the temperature of B is somewhat less than that of A. Similarly the temperature of C is lower than that of B, and so on—the temperature falls off as we advance along the bar.

***Conduction* is a process by which heat passes from a hotter to a colder part of a body, or from a hot body to a colder body in contact with it. There is no transference of the portions of the body lying between the hotter and colder part, but these are all raised in temperature.**

81. Comparison of Conductivities. For this and other experiments it is very convenient to use sensitised paper, which consists of blotting paper which has been soaked in a solution of cobalt chloride and then carefully dried. Such paper is white when cold, but turns green when gently heated.

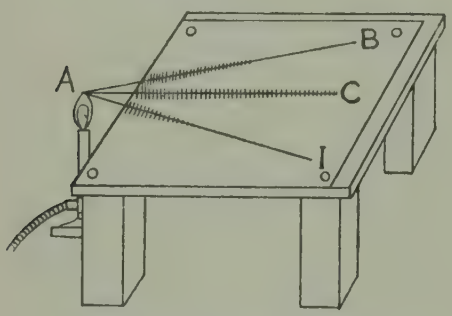

Fig. 36.
Comparison of Conductivities.

To compare the conductivities of say, copper, brass and iron, wires of these materials, similar in length and diameter, are arranged as shown in fig. 36, so that from a common centre A they spread out fan-wise over a sheet of sensitive paper stretched out on a drawing board. At A each wire is turned down to form a short right-angled bend, and the ends are heated by a small bunsen flame. As the heat is conducted along the wires, the paper beneath becomes green. When the green portion has extended as far as it will, the length of each green strip is measured. It can be shown that the conductivities are in the ratio of the *squares* of these lengths. Thus if the green portion extends 10.0 inches along the copper wire, 5.7 inches along the brass, and 4.5 inches along the iron, $C_c : C_b : C_i = 10^2 : 5.7^2 : 4.5^2 = 4.93 : 1.60 : 1$.

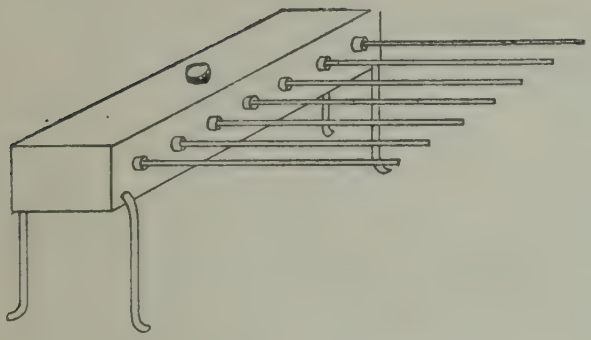

Fig. 37.
Comparison of Conductivities.

Another method of making the comparison is first to cover the rods with a thin coating of paraffin wax, and then place them with their ends dipping into a trough of boiling water as shown in fig. 37. As the heat is conducted along the rods the wax melts, and the distance is measured for each rod up to the point at which the wax just ceases to be melted, the results being worked out as before.

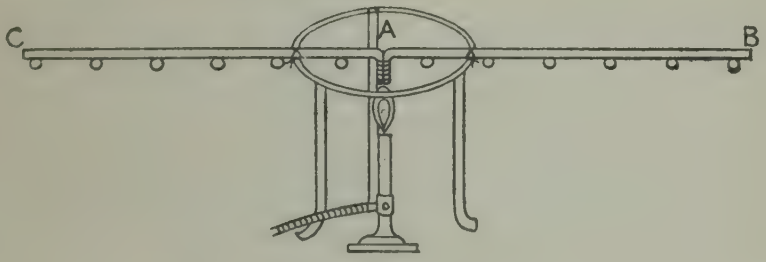

Fig. 38. Conductivity and Specific Heat.

82. Conductivity and Specific Heat. In fig 38 let AB and AC be thin rods (of equal length and diameter) consisting of lead and iron respectively. A number of small wooden balls are lightly attached to them by means of wax, and the ends of the wires are then equally heated at the junction A by means of a small bunsen flame, or otherwise. It is found that :—

(i.) Balls begin to drop off the lead rod first.

(ii.) In the long run, more balls drop off the iron rod.

The explanation is, that the specific heat of lead (.031) is only about a quarter of that of iron (.114). Accordingly, although less heat passes along the lead than along

the iron, this smaller amount of heat raises it to a higher temperature, and balls soon begin to drop off. As time goes on, however, the larger amount of heat passing along the iron makes its effect felt, with the result mentioned in (ii.) above.

At first the temperature of each point along either bar is rising, and the state of the bar is said to be *variable*. A stage is soon reached, however, at which the temperature of any given point on the bar is constant, and the state of the bar is said to be *steady*. It is during the variable state that specific heat enters into the question.

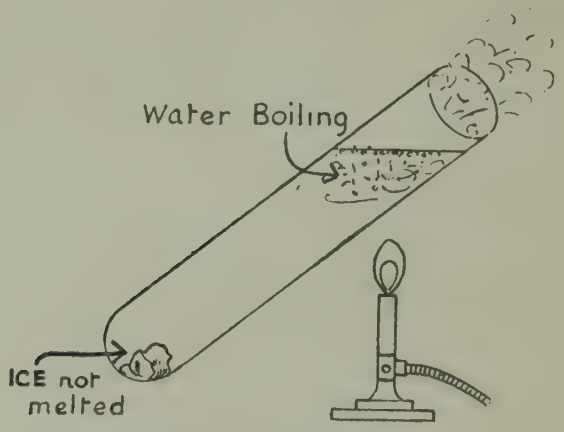

Fig. 39. Water a Bad Conductor.

83. Conductivity of Liquids. A well-known experiment to show the bad conductivity of water is carried out as follows. A test-tube is about two-thirds filled with water, and a piece of ice is sunk in the water by wrapping round it a short length of heavy wire. The water is now boiled at the top, and it is found that while this is taking place the ice remains unmelted at the bottom.

[Q.—Why is this experiment never carried out by allowing the ice to float on the top and heating the test-tube at the bottom ?]

To compare the conductivities of two liquids (e.g., water and mercury) we may use the apparatus sketched in fig. 40. The liquids are placed in two similar test-tubes, round each of which has been bound a strip

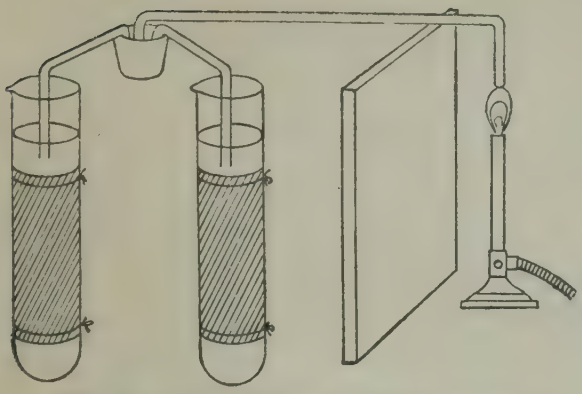

Fig. 40. Conductivities of Liquids compared.

of sensitive paper. Two thick copper wires of equal length dip into a crucible of hot mercury, and by this means the upper parts of the two liquid columns are equally heated. The mercury is kept hot by a small bunsen burner, a screen being used to prevent heat being radiated from the bunsen to the mercury and water.

It is found that the green area spreads a long way down the paper behind the mercury, but only for a very little way down the other. Thus mercury is a much better conductor of heat than water.

Other liquids may be similarly compared.

84. Conductivity in Gases. It is difficult to make direct measurements of conductivity in gases, but we may conclude that gases are very bad conductors of heat, because substances which have a lot of air or other gas entangled among them (e.g., cotton wool) are invariably bad conductors.

85. Uses of Bad Conductors. These are very numerous and only a few can be indicated here.

(i.) Clothing. This reduces the rate at which heat from the body can escape into the air. Loosely woven clothing is "warmer" because of the greater amount of air entangled among it.

(ii.) To pick up a hot object (e.g., the handle of a kettle) we use a bad conductor, such as flannel.

(iii.) Ice is kept in flannel or sawdust, so that as little as possible of the heat of the air may reach it.

86. Uses of Good Conductors.

(i.) Kettles, saucepans, boilers, etc., are made of some good conducting material, so that the heat of the fire may easily reach the water inside.

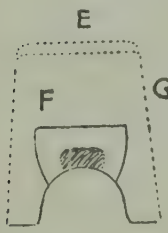

Fig. 41.

(ii.) The Davy Safety Lamp depends for its action on the presence of copper gauze, which acts as a good conductor. Suppose F is the flame of a miner's lamp, enclosed in G (consisting of wire gauze), outside which is E, some of the explosive gas which is often found in mines. Now the gas can, of course, pass through the gauze, and it may burn just round the flame, F, giving it a characteristic blue cap with which the experienced miner is familiar. Not much heat, however, will pass from F to E, because it will be conducted away by the gauze which lies between. The result is that the gas outside G does not reach its "ignition point," i.e., the temperature at which it will explode.

The point is easily illustrated by putting a piece of wire gauze horizontally a few inches above a bunsen burner, turning on the gas and applying a light above the gauze. The gas takes fire above the gauze, but not below. Similarly the gas may be lighted below the gauze without the combustion spreading to the gas above (fig. 42).

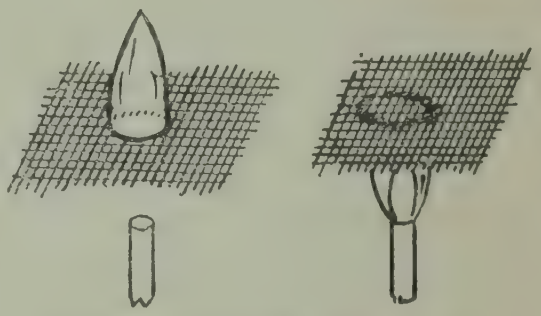
Fig. 42. Action of Wire Gauze on Flame.

[Q.—It is often found that after a little while combustion *does* extend to the other side of the gauze. Why? Can you suggest how this might be prevented?]

87. Convection. When a kettle is heated over the fire, the heat passes (by conduction) through the metal bottom of the kettle and (again by conduction) heats the layer of water in contact with it. As this water is heated it expands, its density becomes less, and therefore it rises to the surface, its place being taken by cold water, which is heated in its turn. Thus the water soon becomes heated throughout.

***Convection* is a process by which heat is carried from one part of a liquid or gas to another by the actual transference of the heated portions of the fluid.**

88. Experiments illustrating Convection.

(i.) If a small crystal of potassium permanganate is placed at the bottom of a flask of water and the latter is then gently heated, the movement of the water can be traced for some time by means of the coloured solution (fig. 43).

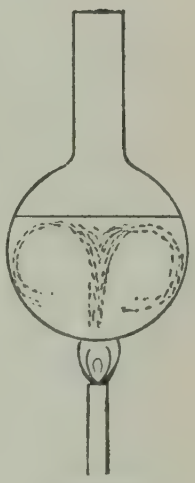

Fig. 43. Convection in a Liquid.

(ii.) A chalk box, with a glass front and two "chimneys" fitted, is arranged as shown in fig. 44, with a candle lighted under one of the chimneys. If some smouldering paper is held above the other one the smoke is seen to take the direction shown.

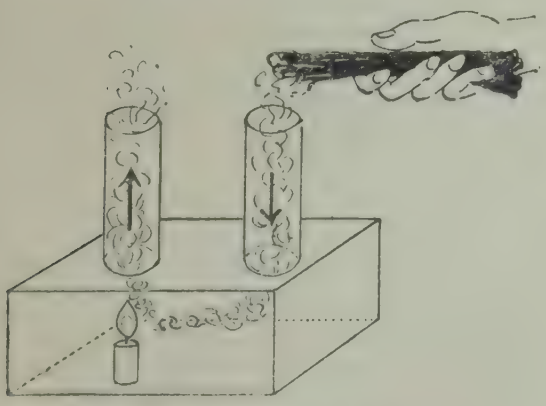

Fig. 44. Convection in a Gas.

This principle was formerly used in the ventilation of mines.

(iii.) The experiment described in the next paragraph gives an excellent illustration of convection in liquids.

89. Heating of Buildings. A common method of heating buildings is illustrated by the apparatus shown in fig. 45. Water is heated in the flask A, which communicates as shown with B, a wide test-tube from which the bottom has been cut off. In B are a few crystals of potassium permanganate, which give a solution with a strong violet colour.

On heating, the water rises by convection up the tube C, and the flask is kept full by a return current down DE, the latter being coloured by the crystals.

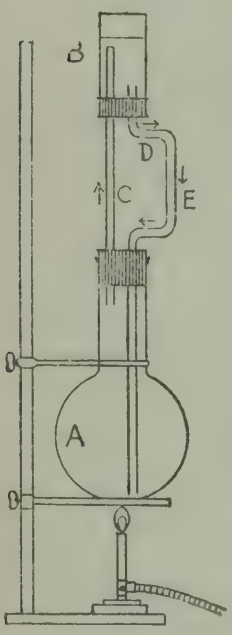

Fig. 45.

In the heating of buildings, we have a boiler in the basement heated by a furnace, just as in the experiment the flask is heated by the bunsen burner. The pipes arranged round the various rooms are often expanded into "radiators," which give off more heat because they present a bigger surface. Instead of the tub B, we have an open tank with a ball and cock arrangement, so that when the water falls below a certain level fresh water runs in from the cistern. Thus the tank serves, (i.) to replace water lost by evaporation, leakage, etc. (ii.) to afford room for expansion. With a completely closed circuit, there would be a burst as soon as the water in the system expanded under the action of heat.

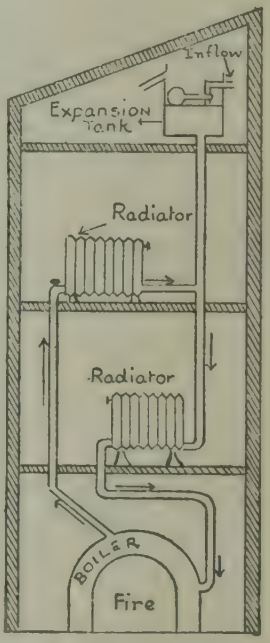

Fig. 46. Heating by Hot-water Pipes and Radiators.

In diagrammatic form the arrangement is shown in fig. 46.

90. Applications and Consequences of Convection.
(i.) The *heating of buildings* and *an old method of ventilating mines* have already been discussed, and so has the *heating of liquids* (§87).

(ii.) *Ventilation of Rooms.* The heated air over the fire rises up the chimney, and fresh air is drawn in through the door, windows, etc. Thus a fire serves to ventilate a room as well as to warm it.

(iii.) *Winds are a result of Convection.* This is very clearly seen by the seaside, where we usually have a sea breeze blowing during the day and a land breeze during the night. During the day the land, on account of its lower specific heat, becomes warmer than the sea. Hence the air above the land becomes warm and rises, its place being taken by cool air from the sea. During the night the land cools quickly, and the effect is reversed.

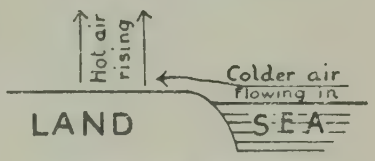

Fig. 47.
Production of a Sea-breeze.

91. Results of Impeding Convection. When a liquid is being heated in a glass (or other) vessel, the convection currents in the liquid cause this heat to be fairly evenly distributed over the sides of the vessel. In the case of a glass vessel this is important, because unequal heating would result in unequal expansion, and the vessel would crack.

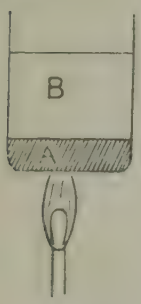

Fig. 48.

Now suppose we intend to dissolve a large quantity of salt or other substance, and that we put the substance in all at once (A in fig. 48) and cover it with the water B. Convection is obviously prevented in the region A, and the bottom of the beaker becomes hotter than the upper portion, with the result that the beaker will probably crack.

[Q.—How *would* you dissolve a large quantity of salt, etc. ?]

In very "thick" liquids convection is greatly impeded, with the result that one part may become much hotter than another. This fact has often to be taken into account.

[Q.—Why does jam "burn" unless it is constantly stirred when being made ?]

[Q.—Why is a plate of porridge often quite cool on the surface and yet very hot inside ?]

92. Radiation. It was pointed out in §79 that heat may travel from one point to another without either convection or conduction coming into play. Heat from the sun cannot reach the earth by either of these processes, because for the whole of the distance separating the earth and the sun (except for just a few miles near the earth) there is no air or other material medium. The process by which heat thus travels across space is known as *Radiation*. We shall consider a few of its characteristic features before giving a formal definition.

(i.) *The intervening medium (if any) is not heated.* If the hand is held in front of a hot fire for a little while, and then a screen is suddenly interposed, the cooling effect is felt *immediately*. If the heat were stored up in the intervening air, the hand would continue to feel hot for some time after the screen had been interposed.

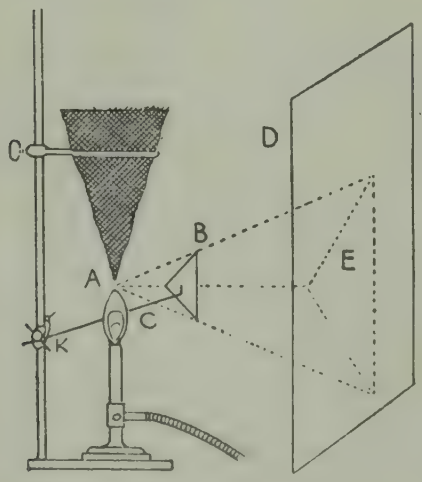

Fig. 49.
Heat is Radiated in Straight Lines.

(ii.) Heat is radiated *with great speed*. Careful observations during an eclipse show that the temperature begins to fall as soon as the sun's light ceases to be received. Hence heat must be radiated with the same speed as light.

(iii.) Heat is radiated *in straight lines*. In fig. 49, A represents a "point-source" of heat, made by folding a piece of wire-gauze into the

shape of a cone and heating the narrow end with the tip of a bunsen flame. (In practice only a very rough approximation to a point-source can be obtained.) B is a small flat piece of metal of some definite geometrical shape, triangular for instance, held in position by the wire C to which it is soldered, and the other end of which is fixed to the retort-stand at K by means of a collar. D is a screen covered with heat-sensitive paper. The distance from A to the metal is short, about an inch, and from the metal to the screen is about another inch.

On heating A, D soon becomes green except for a white patch, E. On substituting a very small electric light bulb for A, and darkening the room, the shadow obtained on D is found to coincide with the white patch as nearly as could be expected considering the imperfection of the point-source of heat.

Hence the rays of heat must have travelled in the same direction as the rays of light, i.e., in a straight line (§97). We are now in a position to define radiation.

Radiation is a process in which heat passes from one place to another in straight lines, with great speed, and without heating the medium through which it passes.

93. Radiating Powers of Different Surfaces. Two similar calorimeters (or cocoa tins, mustard tins, etc., will do quite well) are taken. A is brightly polished, but the surface of B is made a dull black by holding it in the smoke from burning camphor or turpentine. Each tin is closed by a cork through which a thermometer passes, and equal quantities of hot water are poured into the two tins. The time taken for the water to cool through a given range of temperature, say from 80°

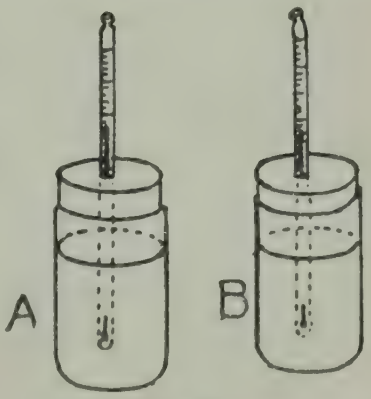

Fig. 50

to 50°, is carefully noted, and it is found that cooling takes place more quickly in the blackened calorimeter. Other experiments show that the result is a general one, i.e., other things being equal, a dull surface radiates heat more rapidly than a bright one.

Hence, if it is required to reduce loss of heat to a minimum, the surface of the body concerned should be brightly polished. Thus a silver tea-pot, or a brightly polished copper kettle, keeps hot for a long time. For the same reason, the surface of the inner vessel of a thermos flask is covered with a thin coating of polished silver. On the other hand, hot-water pipes and radiators are usually painted a dull black.

94. Absorbing Powers of Different Surfaces. Let the calorimeters of the last experiment be filled with equal quantities of *cold* water, and placed at equal distances from the open mouth of a muffle furnace. It will be found that the water in the blackened calorimeter becomes hot much sooner than that in the bright one. Thus the dull surface, which was found to radiate heat more rapidly than the bright one, also absorbs heat more rapidly. *Good absorbers are also good radiators,* and vice versa.

95. Theory of Radiation. In §38 it was stated that "heat is a form of energy." Let us examine this statement a little more closely.

If a closed basket packed with cups and saucers were allowed to fall from a height, it would be in a state of rapid motion just before it reached the ground. It would therefore, *considered as a whole,* possess kinetic energy. On striking the ground, the basket comes to rest. Its kinetic energy, considered as a whole, has disappeared. Much of this, however, is taken up by the contained parts, and these are set into violent motion, with disastrous consequences.

This rough illustration will help us to understand why a bullet becomes hot when it strikes a target. The kinetic energy possessed by the bullet as a whole is

passed on to the particles ("molecules") composing it. The molecules must be in a state of violent motion; the bullet feels hot. Hence it is easy to realise that the sensation of "hotness," i.e., temperature, is connected with molecular movement. There is good reason to believe that, except at absolute zero, molecules are always in a state of vibration, and a rise of temperature is the outward and visible sign of an increase in the frequency of vibration.

The heat from the sun reaches us somewhat as follows. It is believed that all space is filled by a curious substance called the *ether*. The molecular vibrations of the heated substance set up waves (of corresponding frequency) in the ether, and these waves are transmitted to the earth. Then, just as in the first place the ether waves were set up by molecular vibrations, so, on reaching the earth, molecular vibrations are set up by the ether waves —the earth becomes hotter.

The ether waves reaching us from the sun are of all degrees of frequency, and corresponding wave-length (high frequency goes with short wave-length and vice versa). Comparatively long waves give rise to a sensation of heat, while shorter waves also give rise to the sensation of light. Then, of the light waves, the longest give rise to the sensation of red and the shortest to that of violet, other colours lying in between. There are shorter waves than the violet (the "ultra-violet") which produce no light effects, though they are able to assist in bringing about many chemical changes. The band of colour which under suitable conditions may be obtained from sunlight, is known as the *visible spectrum*. On either side of this we have an invisible spectrum, where those rays are received which are either too long ("infra-red") or too short ("ultra-violet") to produce colour effect.

There is plenty of evidence to show that radiant heat is similar in many respects to light. Thus:—

(i.) It has already been shown that it travels in straight lines, and that

(ii.) It travels with the same speed as light ;

(iii.) It is reflected and refracted in accordance with the same laws as light ;

(iv.) It is readily absorbed by dark surfaces.

96. Diathermancy. We are all very familiar with the fact that light passes through some substances much more easily than others—through clear glass, for instance, much more easily than through smoked glass. We should say that the two are not equally *transparent*. Now it can be shown (though not with very elementary apparatus) that some substances allow *heat* to pass through them more easily than others. For this "heat transparency," the word *diathermancy* is used. Thus a sheet of glass is far more diathermanous than one of iron.

Further, some glass is transparent only to red light, other glass to blue light, etc., i.e., the transparency of glass depends on the wave-length of the light falling upon it. "Clear" glass is transparent to light of all wave-lengths.

Similarly, a substance which is diathermanous to heat waves of a particular length may not be diathermanous to heat waves of a very different length. An important case is that of glass. Heat waves from a body at a very high temperature such as the sun (i.e., the shorter heat waves), pass through it readily, but if the radiating source is at a lower temperature (e.g., warm soil) the heat waves are unable to pass through. Here we have the principle of the greenhouse. The heat from the sun is able to enter freely and warm up the soil, wood, plant-pots, etc. These then begin to radiate their heat, but this is unable to pass through the glass, and so the temperature of the greenhouse begins to rise.

QUESTIONS ON CHAPTER V.

1. One way of keeping a liquid cool on a hot day is to wrap up the containing vessel in a damp cloth ; another is to put it in a vacuum flask with silvered sides. Explain the effectiveness of each method.—*M.L.L.S.B.* 1926.

2. In what ways does a hot object lose its heat to its surroundings ? Describe methods of hastening the cooling process. —*Lond.* 1925.

3. A vessel of hot water can lose heat in four different ways Explain the methods by which these losses may be diminished. —*Lond.* 1923.

4. Two metal bars, A and B, of the same size but of different material, are coated with equal thicknesses of wax and placed each with one end in a hot bath. It is noticed that at first the wax on A melts at a greater rate than that on B, but that when a steady state has been reached, a greater length of wax has been melted on B than on A. Explain this.—*Camb.* 1925.

5. A metal rod is coated with wax and one end is placed in a bath of boiling water. What are the properties which determine, (a) the rate at which the wax melts along the rod, (b) the distance over which it will melt ?—*Camb.* 1924.

6. A piece of paraffin wax is placed in a test tube of water, and it melts quickly when a flame is applied at the bottom of the tube. If, however, the wax is fixed at the bottom of the tube and the flame applied near the top, a considerable time elapses before the wax begins to melt. Account for this difference in time and describe how the heat reaches the wax from the flame in each case.—*Lond.* 1924.

7. If a piece of fine wire gauze is placed over a gas jet, and the gas is then turned on and lighted above the gauze, the flame does not strike back through the gauze. Why is this ?

Give an account of the construction and use of a Davy Safety Lamp for miners —*Oxf.* 1924.

8. A large piece of ice is placed in a tall jug of water at the temperature of the room. Describe and explain the processes by which the water in the jug is cooled.—*O.C.* 1921.

9. How is heat transmitted through gases ? Describe experiments in illustration.—*Camb.* 1923.

10. A kettle of water is suspended over a glowing fire. Describe how the heat is transferred from the fire to the water.

The handle of a kettle is made of stout iron wires plaited closely together at the ends but expanded into an open spiral in the centre. Explain the reasons for this arrangement.—*Lond.* 1923.

11. Explain why the radiators in a building heated by hot water are usually dark in colour, unpolished and shaped as they are. Describe any experiments you have seen performed which confirm the reasons you have given.—*M.L.L.S.B.* 1923.

12. Explain fully the physical principles involved in the heating of a building by steam pipes.—*Camb.* 1922.

13. Point out very carefully all the various phenomena which come into action when a damp cloth held in front of a fire begins to "steam."—*Lond.* 1921.

14. Describe how you would compare the thermal radiating powers of different surfaces. What evidence have we for stating that good radiators are also good absorbers of heat ?—*Camb.* 1923

15. Give a brief account of the properties of radiant heat. Mention three facts illustrating the similarity in character between light and radiant heat.—*Camb.* 1925.

16. Give an account of the nature of heat radiation. How does it differ from visible light ?

Explain why the sun makes the air inside a greenhouse hotter than that outside.—*Oxf.* 1924.

Part II.
LIGHT.

Chapter VI.

PROPAGATION, PHOTOMETRY, REFLEXION AT PLANE SURFACES.

97. Light Travels in Straight Lines. When light is travelling through a homogeneous medium, i.e., through a medium which is of the same material and the same density in all its parts, it travels in straight lines. We have evidence of this in the appearance of a pencil of light from a cinema lantern, or in the fact that we are unable to see round a corner. The fact may be proved experimentally as follows.

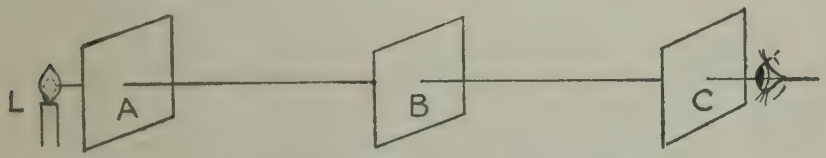

Fig. 51. Light Travels in Straight Lines.

Three small sheets of cardboard are clamped so as to be roughly parallel to one another, the distance between them being about a yard. Small holes, A, B, C, are bored in the centre of each sheet, and a piece of thread is passed through. The sheets are so adjusted, that when the thread is pulled tight it just passes through the middle of the holes without catching the edges. A, B and C are evidently now in the same straight line.

A lighted candle is now placed at L, just behind A. By an observer looking through C, the flame can be seen along the line CBA, but if one of the pieces of cardboard, B, is slightly moved, it becomes invisible.

98. The Pinhole Camera. This depends for its action on the fact that light travels in straight lines. A box,

A, slides inside another box, B. A is partly open at one end, so that an eye (E in fig. 52) may look through the aperture. The other end of A is closed with tissue paper or (better) with a screen of ground glass. B, of course, is closed on one side by the ground glass screen. The other end is closed by a piece of cardboard pierced with a small hole, H.

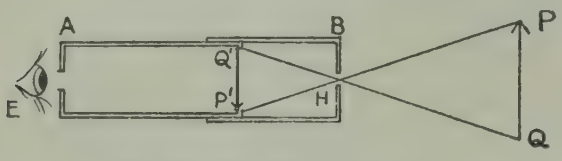

Fig. 52. The Pinhole Camera.

On pointing the apparatus towards a bright object, such as a lighted candle (represented by PQ in the figure), an inverted image is observed on the screen. The reason for the inversion will be clear from the figure —the rays of light cross at H.

Suppose that, instead of having only one hole at H, we have several, say three, very close together. The result will evidently be to give us three images of PQ very close together. In practice these will appear as a single image with a somewhat blurred outline.

We can now see what would be the effect of enlarging the hole H. The large hole may be regarded as made up of several small ones, and so the image will have a blurred outline. On the other hand, the image will be brighter, for the simple reason that more light can pass through a large hole than through a small one.

The pinhole camera has been successfully used in photography, a sensitive plate taking the place of the ground-glass screen.

99. Shadows. It will be convenient to distinguish between the shadows cast when the source of light is :—

(1.) A point.

(2.) Smaller than the intervening object, though not actually a point.

(3.) Larger than the intervening object.

The three cases are illustrated by figs. 53—55. For simplicity, and because of its importance in connection

with eclipses, a sphere has been selected as the intervening object in each case, and as the source of light in figs. 54 and 55.

In all cases there is a cone-shaped region of complete shadow known as the *umbra*. Outside this, except in

Fig. 53. Point Source of Light.

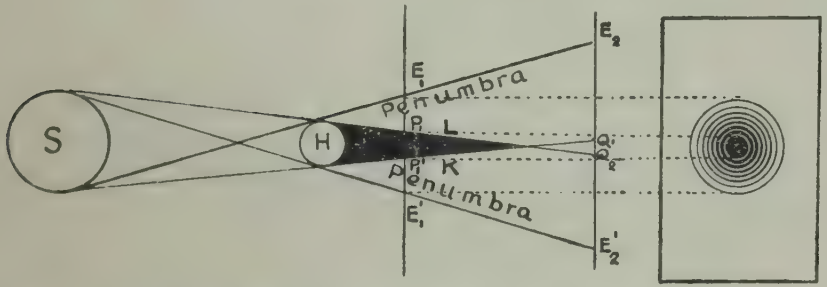

Fig. 54. Source of Light larger than intervening object.

Fig. 54a.

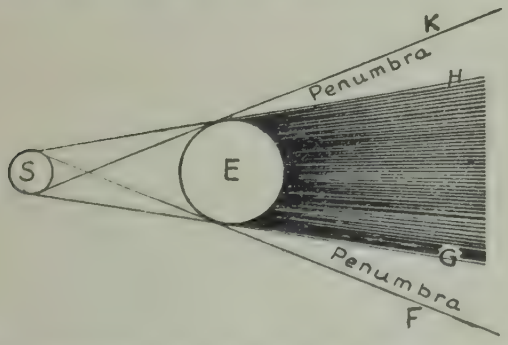

Fig. 55. Source of Light smaller than intervening object.

the case where the source of light is a point, there is a region of partial shadow, the *penumbra*. In the penumbra there is a gradual transition from complete darkness on the side where it borders on the umbra, to complete light on the other side. Thus in fig. 54, if a screen is placed at $E_1 E_1'$, the shadow will be as indicated in fig. 54a, which shows a front view instead of a side view.

100. Eclipses.

(1.) *Of the Sun.* In fig. 54, let S represent the sun, H the moon, and E_1 E_1' the earth. To a person placed between P_1 and P_1', the sun will be totally eclipsed. This region of total eclipse is a comparatively narrow one, much narrower than the figure would suggest, but as the moon advances in its orbit round the earth, the region of total eclipse will move too. A person between P_1 and E_1 (or P_1' and E_1') is in the penumbra, and to him the eclipse is partial. Near P_1 (or P_1') he would see only a narrow crescent of light, while near E_1 (or E_1') he would see nearly the whole of the sun's disc.

Now the moon's orbit round the earth is not a circle, but an ellipse. Hence the earth is sometimes further away from the moon than is represented by the position E_1 E_1'. Suppose E_2 E_2' represents this more distant position. Here there is no region of total eclipse. A person between Q_1 and Q_2 sees the sun eclipsed except for its outer ring—he sees, in fact, what is known as an **annular eclipse.** Between Q_1 and E_2 (or Q_2 and E_2') there is a partial eclipse of steadily diminishing completeness.

(2.) *Of the Moon.* In fig. 54, let S and H now represent the sun and earth respectively, and let E_1' K L E_1 represent a portion of the moon's orbit. At E_1' the moon enters the penumbra, but there is not much loss of light until it draws near to K. Beyond K it is within the umbra, and an observer on the earth sees a deep shadow with a circular edge gradually extending over the moon's surface. The shadow may cover the moon completely, in which case the eclipse is said to be *total.*

[Q.—Why do we not always have an eclipse of the sun at "new moon" and an eclipse of the moon at "full moon"?]

101. Illuminating Power. It is often necessary to determine the illuminating power of a given source of light. Evidently we need some standard, and the one employed is the illuminating power of the **standard**

candle, a standard candle being defined as **a sperm candle weighing one-sixth of a pound and burning away at the rate of 120 grains per hour.**

Thus an electric light bulb which has an illuminating power equal to that of 50 standard candles, is said to be of 50 candle power.

102. Intensity of Illumination. From what has just been said, it will be evident that *illuminating power* has reference to the *source of light.* It may be measured by the total light energy emitted by the source in one second. We must now consider the surface on which the light falls. Suppose a surface of area S is uniformly illuminated, and that E units of light energy cross the surface at right angles per second. Thus the amount received per unit area, per second, is E/S, and this quantity is known as the *intensity of illumination of the surface.* If the surface is not uniformly illuminated, then the intensity of illumination at any particular point is found by working out the ratio e/s, where s is a very small area surrounding the point, and e is the quantity of light energy which crosses that very small area per second at right angles.

103. Law of Inverse Squares. Suppose S is a small source of light of illuminating power E, i.e., giving off E units of light energy per second, and consider two imaginary spherical surfaces in the surrounding medium, having S as their common centre, the radii of the surfaces being R_1 and R_2 respectively.

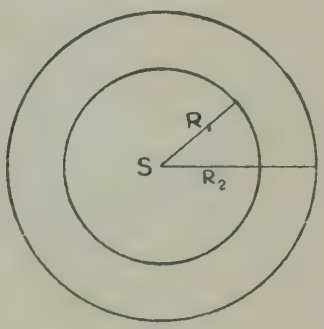

Fig. 56.
Law of Inverse Squares.

The area of the smaller surface is $4\pi R_1^2$, and that of the larger surface is $4\ \pi\ R_2^2$. From the definition just given, we see that I_1, the intensity of illumination of

the smaller surface, is $\frac{E}{4\pi R_1^2}$ and that I_2, the corresponding quantity for the larger surface, is $\frac{E}{4\pi R_2^2}$

$$\text{Therefore } \frac{I_1}{I_2} = \frac{E}{4\pi R_1^2} \div \frac{E}{4\pi R_2^2} = \frac{R_2^2}{R_1^2}$$

Hence we have the important result that **the intensity of illumination at any point is inversely proportional to the square of its distance from the source of light.** This result is known as the ***Law of Inverse Squares.***

The intensity of illumination is also, of course, **proportional to the illuminating power of the source,** e.g., at a given distance, two candles would evidently give twice the intensity of illumination that one candle would give.

Example 28. Compare the intensity of the illumination produced by a 50 c.p. lamp at a distance of 6 ft. with that produced by a 100 c.p. lamp at 8 ft. At what distance should the latter be placed so as to give the same intensity of illumination as the former ?—*Lond.* 1924.

(i.) Suppose intensity of illumination of 1 c.p. lamp at distance 1 ft. is represented by x.

Then intensity of 50 c.p. at 1 ft. is represented by $50x$.

$\therefore$ intensity of 50 c.p. at 6 ft. is represented by $\frac{50x}{6^2}$

Similarly intensity of 100 c.p. at 8 ft. is represented by $\frac{100x}{8^2}$

$$\therefore \frac{\text{Intensity in 1st case}}{\text{Intensity in 2nd case}} = \frac{50x}{6^2} \div \frac{100x}{8^2} = \frac{8}{9}$$

(ii.) Suppose required distance is d ft.

Then as before,

Intensity of 50 c.p. at 6 ft. is represented by $\frac{50x}{6^2}$

Intensity of 100 c.p. at d ft. is represented by $\frac{100x}{d^2}$

These are to be equal,

$\therefore \frac{50x}{6^2} = \frac{100x}{d^2}$ giving $d = \sqrt{72}$ ft. or 8.48 ft.

104. Photometry. By applying the Law of Inverse Squares, we have a means of finding the candle power of a given source of light. The apparatus used in such a determination is known as a *photometer*, the process itself being known as *photometry*. We shall describe the use of, (i.) Rumford's Shadow Photometer, and (ii.) Bunsen's Grease-Spot Photometer.

105. Rumford's Shadow Photometer. A screen, E, is covered with a sheet of unglazed white paper, and a vertical rod (the rod of a retort stand does very well) is placed just in front of it. Suppose we wish to compare the illuminating powers of an electric light bulb, A, and of a candle, B.

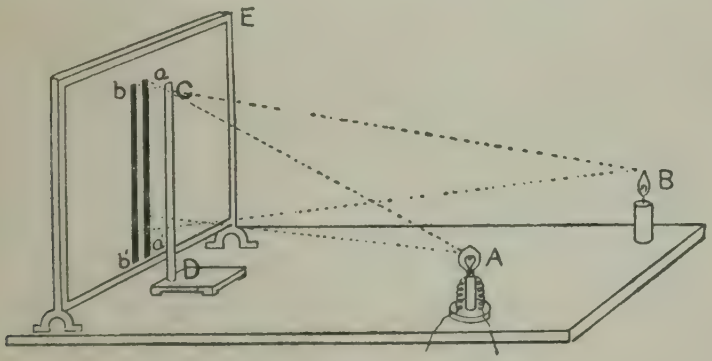

Fig. 57. Rumford's Shadow Photometer.

The two sources of light are so placed that two well-defined shadows of the rod, aa′ and bb′, are obtained side by side. We must next arrange that these shadows shall be of equal depth.

Now bb′ is evidently illuminated by A, while aa′ is illuminated by B. If, therefore, bb′ is too dark, we move A nearer to the screen and vice versa. Suppose that shadows of equal depth are obtained when the distances of A and B from the screen are d_a and d_b respectively. Then if I_a and I_b are the respective illuminating powers of the electric light and of the candle, we shall have $\frac{I_a}{I_b} = \frac{{d_a}^2}{{d_b}^2}$ Why ?

The intensity of illumination produced by A is (i.) proportional to its illuminating power, (ii.) inversely proportional to the square of its distance from the screen (Law of Inverse Squares).

$\therefore$ Intensity of illumination $= \frac{K\,I_a}{d_a^2}$ when K is some constant.

Similarly, intensity of illumination produced by B $= \frac{K\,I_b}{d_b^2}$

But the intensity of illumination produced by A is equal to that produced by B.

$$\therefore \frac{K\,I_a}{d_a^2} = \frac{K\,I_b}{d_b^2} \qquad \therefore \frac{I_a}{I_b} = \frac{d_a^2}{d_b^2}$$

If B is a standard candle, then $I_b = 1$, and $I_a = \frac{d_a^2}{d_b^2}$

Example 29. In using Rumford's Shadow Photometer, shadows of equal depth are obtained when a standard 4 c.p. lamp is at a distance of 3 ft. and an electric light is at a distance of 8 ft. Find the c.p. of the electric light.

If x stands (as before) for the intensity of illumination of a 1 c.p. lamp at distance 1 ft., then intensity of illumination of 4 c.p. lamp at 3 ft. is $\frac{4x}{3^2}$

And intensity of illumination of electric lamp at 8 ft. is $\frac{cx}{8^2}$ where c = required candle-power.

But these intensities are equal.

$$\therefore \frac{4x}{9} = \frac{cx}{64}$$

Cancelling x, we have $9c = 256$, $\therefore c =$ **28.4**

(or we could have applied the formula $\frac{I_a}{I_b} = \frac{d_a^2}{d_b^2}$ getting $\frac{c}{4} = \frac{8^2}{3^2}$ or $c = 28.4$ as before.)

106. **Bunsen's Grease-Spot Photometer.** This consists essentially of a piece of unglazed paper, suitably mounted,

with a spot of grease in the centre. When the paper is equally illuminated on both sides, the grease-spot cannot be distinguished from the rest of the paper. The two sources of light are adjusted in position, one on each side of the photometer, until the grease-spot cannot be distinguished.

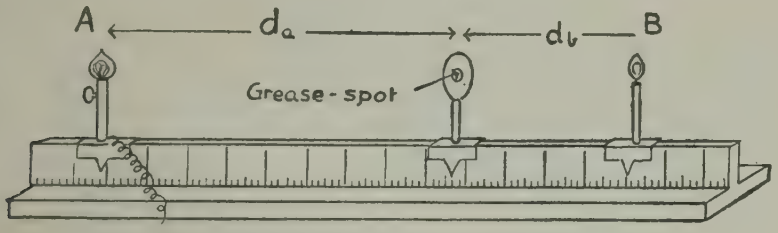

Fig. 58. Bunsen's Grease-spot Photometer.

Suppose source A is then at a distance d_a, and B at a distance d_b. Then intensity of illumination due to source A is $\frac{K I_a}{d_a^2}$ and intensity due to source B is $\frac{K I_b}{d_b^2}$

These are equal.

$$\therefore \frac{K I_a}{d_a^2} = \frac{K I_b}{d_b^2}$$

$$\therefore \frac{I_a}{I_b} = \frac{d_a^2}{d_b^2}$$

Precautions to secure a good result.

(i.) A and B must be the *only* sources of light, i.e., the room must be darkened.

(ii.) We should take the mean of at least four results. (1) When grease-spot disappears looked at from the side of A ; (2) the same from the side of B ; (3) and (4),—the first two results repeated, but with the screen turned through 180°.

Example 30. Two lamps of 16 c.p. and 24 c.p. are 5 ft. apart. A screen is placed between them 2 ft. from the weaker lamp. Compare the illuminations of the two sides of the screen. Where must it be placed to be equally illuminated on both sides ?—*Lond.* 1924.

(i.) Let x = intensity of illumination produced by light of 1 c.p. at distance of 1 ft.

Weaker lamp (16 c.p.) is 2 ft. from screen.

$\therefore$ it gives intensity of illumination $\frac{16x}{2^2} = 4x$

24 c.p. lamp is 3 ft. away.

$\therefore$ it gives intensity of illumination $\frac{24x}{3^2} = \frac{8x}{3}$

$\therefore$ required ratio $= 4x \div \frac{8x}{3} = \mathbf{\frac{3}{2}}$

(ii.) Suppose that for equal illumination, screen must be d ft. from 16 c.p. lamp and therefore $(5-d)$ ft. from 24 c.p. lamp.

Intensity of illumination due to 16 c.p. lamp is $\frac{16x}{d^2}$

Intensity of illumination due to 24 c.p. lamp is $\frac{24x}{(5-d)^2}$

$\therefore \frac{16x}{d^2} = \frac{24x}{(5-d)^2}$

Cancelling x, and solving the quadratic in d, we obtain $d = 2.25$ ft.

$\therefore$ screen must be **2.25 ft.** from the weaker lamp.

107. Illustration of Law of Inverse Squares. Bunsen's Photometer may be conveniently used to illustrate the Law of Inverse Squares. For this purpose, A is made to consist of a compact group of four candles, while B consists of one candle, all the candles being as nearly as possible alike. d_a is then found to be twice d_b.

$\therefore$ *four* candles at a certain distance produce the same intensity of illumination as one candle at half that distance.

$\therefore$ *one* candle at a certain distance produces a quarter the intensity of illumination as one candle at half that distance—a result which is in agreement with the Law of Inverse Squares.

The number of candles may, of course, be varied, enabling us to obtain further illustrations of the Law.

108. Laws of Reflexion.

(1.) **The reflected ray lies in the plane containing the incident ray and the normal.**

(2.) **The angle of reflexion is equal to the angle of incidence.**

Notes.

(a) The "normal" is at right angles to the surface.

(b) To understand the first law, imagine a perfectly flat sheet of cardboard to be so placed as to contain both the incident ray and the normal. Then the same flat surface will also contain the reflected ray.

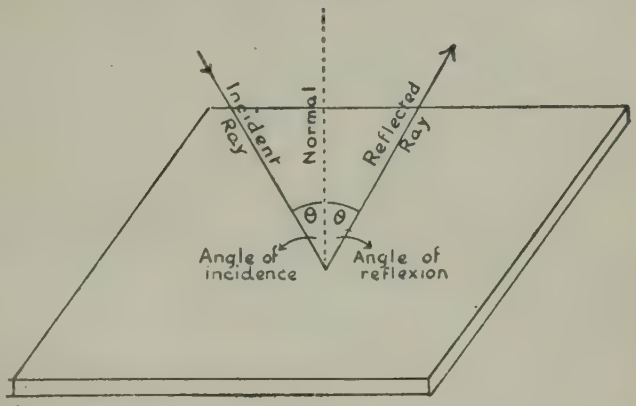

Fig. 59.

Experimental Proof.

AB represents a mirror about 6 ins. by 1 in., resting on one of its long edges on a sheet of paper (in practice it is usually glued to a block of wood to prevent it from falling over).

P_1 and P_2 are two pins, their images being at P_1' and P_2' respectively. Two other pins, P_3 and P_4, are so placed that they are in a straight line with P_1' and P_2'. The normal is drawn, and the angles of incidence and reflexion (indicated in the figure) are measured with a protractor. They should be found to be equal.

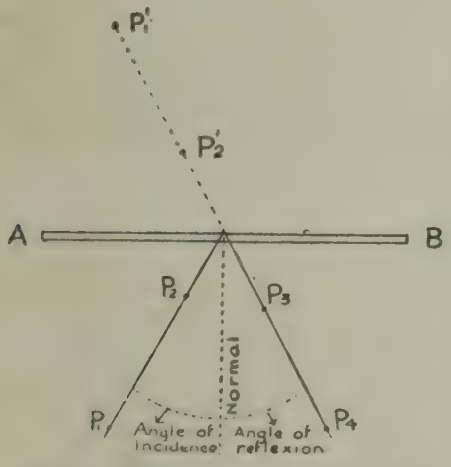

Fig. 60. Proof of Laws of Reflexion.

How does this experiment illustrate the *first* law? Consider a ray of light travelling from the foot of

P_1 to the foot of P_2, and therefore travelling in the plane of the paper. The corresponding reflected ray is that which appears to have come from the foot of P_1' to the foot of P_2', and this line is found by experiment to be in the plane of the paper. Hence the incident ray and the reflected ray are both in the same plane, i.e., the plane of the paper.

The experiment is rather more convincing if carried out with a narrow visible beam of light instead of with pins. This is easily obtained by arranging to make the light of a candle pass through the narrow space between two parallel blocks almost touching one another. If the room can be darkened so much the better, though this is not quite essential. The method of obtaining the angles of incidence and reflexion will be sufficiently clear from fig. 61.

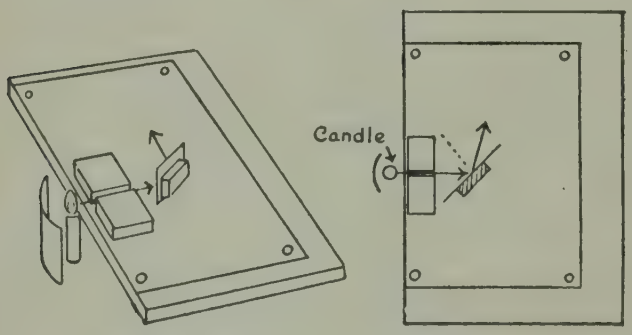

Fig. 61. Proof of Laws of Reflexion.

By tilting the mirror backwards and holding a small set-square against it (the edge of which represents the normal), it is easily shown that the reflected ray, the incident ray, and the normal all lie in one plane.

(The fact that on rotating the mirror through an angle, α, the reflected ray is rotated through an angle, 2α—see next paragraph—may be illustrated with the same apparatus.)

109. Rotation of Mirror. *If a ray of light is reflected from a mirror and the latter is turned through an angle* α, *the reflected ray will be turned through an angle* 2α.

Suppose the ray of light is incident normally on the mirror AB. Then it will be reflected along its own path,

i.e., along DC. Now let the mirror be turned through an angle α into the position A′ B′.

The normal to the mirror will evidently be turned through the same angle α into the position DC′ (for we may think of the normal as a thin wire rigidly attached to the mirror, and such a wire would evidently turn through the same angle as the mirror itself.)

The ray incident along CD will now be reflected along DE, where the angle of reflexion C′DE = angle of incidence C′DC = α.

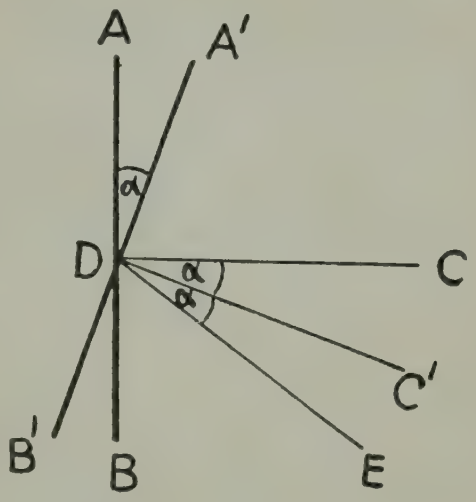

Fig. 62.
Effect of Rotating Plane Mirror.

The reflected ray has clearly been turned through the angle CDE = 2 α.

110. Application to Angle of Prism. The principle just discussed may be used to find the angle of a prism.

The method consists in placing the prism on its triangular end, and drawing two parallel lines, DE and FG, to meet, in H and L, those sides of the triangle which include the angle to be measured. Two pins are placed on one of the lines, at D and E, and two more on the other one at F and G. The ray of light incident along DE is reflected along HK, and we mark the direction of this reflected

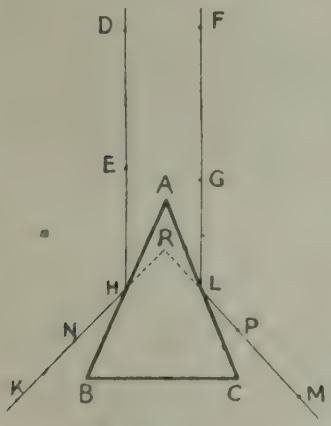

Fig. 63.
Finding Angle of a Prism.

ray by inserting pins at N and K (these two pins and those at D and E will appear to be in one straight line). Similarly we mark PM, the direction along which the incident ray FG is reflected.

Now think of AB as a mirror hinged at A, first reflecting the ray DE along HK, and then after being rotated into the position AC, reflecting the ray (or at least one parallel to it) along LM. The reflected ray has been turned through the angle KRM. Therefore the mirror has been turned through half that angle. Hence the required angle, BAC, is obtained by measuring the angle KRM, and halving it.

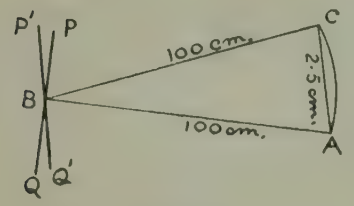

Fig. 64.

Example 31. A horizontal beam of light, AB, falls normally on the centre, B, of a plane mirror, placed in a vertical position. It is required to produce a reflected beam, BC, such that BA = BC = 1 metre and AC = 2.5 cms. Find, approximately, the angle through which the mirror must be rotated.—*Lond.* 1925.

Let PQ represent the mirror, at right angles to AB. Before rotation, AB is reflected along BA. After rotation (into the position P′Q′), AB is reflected along BC. We have AB = BC = 100 cms. and AC = 2.5 cms. (For clearness, AC is much exaggerated in the figure.) The angle ABC in circular measure

$$= \frac{\text{arc AC}}{\text{AB}} = \text{(approximately)}\ \frac{\text{chord AC}}{\text{AB}} = \frac{2.5}{100} = .025.$$

$$\therefore \text{ angle ABC in degrees} = .025 \times \frac{180}{3\frac{1}{7}} \quad (\text{for } \pi \text{ radians} = 180°$$

$= 1.43°$. This is the angle through which the *reflected ray* has been rotated.

$\therefore$ angle through which *mirror* has been rotated $= 1.43 \div 2 =$ **.71°.**

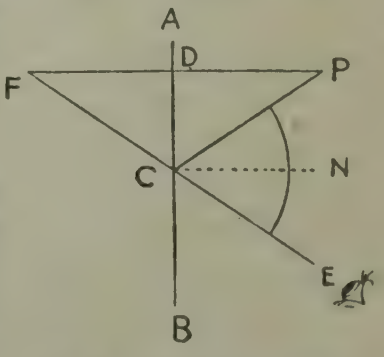

Fig. 65 Position of Image.

111. Position of Image in Plane Mirror. Let P be a point in front of a plane mirror, AB, and let PC be any ray of light proceeding from P and striking the mirror at C. At C draw the normal CN.

PC will be reflected along CE in such a way that angle ECN = angle PCN.

From P draw PD at right angles to the mirror, and let PD and EC when produced meet in F.

Considering the triangles CDF and CDP,

Angle DFC = angle NCE (by parallels).
= angle NCP (law of reflexion).
= angle DPC (by parallels).

Hence in the triangles CDF and CDP we have
angle DFC = angle DPC (just proved),
angle FDC = angle PDC (right angles),
and CD is common to both,

∴ the triangles are congruent, and in particular DF = DP.

Now PCE represents *any* ray proceeding from P and reflected from the mirror. Hence *all* rays proceeding from P will, after reflexion from the mirror, appear to proceed from a point, F, so placed that PF is perpendicular to the mirror and DF = DP.

Thus *the image of a point in front of a mirror is formed at an equal distance behind, in such a position that the line joining object and image is at right angles to the mirror.*

Experimental Proof. A mirror, M, about 6 ins. by 1 in., is placed, edge-on, on a sheet of paper resting on the table as shown in fig. 66. (The block, with saw-cut, merely serves as support.) A pin, P, is placed upright a few inches in front of the mirror, giving an image, P′, behind. We have to fix the exact position of P′. To do this, take a pin, T, sufficiently long for its upper part to be seen above the top of the mirror. Move T about until it coincides with the image, i.e., until T and the image appear to keep together when

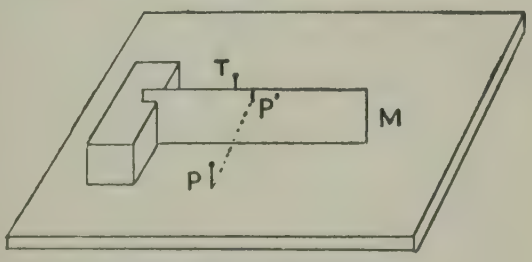

Fig. 66. Position of Image.

the head is moved from side to side. T now gives the position of the image, and a simple measurement will serve to verify the statement made in the last paragraph.

112. Lateral Inversion. Suppose an object, say one shaped like the letter F, is placed in front of a mirror. Each point on the object gives a corresponding image, and the position of each of these point-images is easily determined by the principle of §111. The construction is sufficiently indicated by fig. 67.

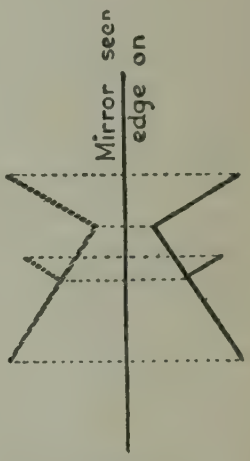

Fig. 67.
Lateral Inversion.

The image is evidently *laterally inverted* with respect to the object, i.e., the left side of the object is the right side of the image and vice versa.

We have a familiar example of lateral inversion when we hold up some writing in front of a mirror.

113. Parallel Mirrors. Let M_1 and M_2 represent two parallel mirrors, L an object placed between them. From L draw a line perpendicular to either of the mirrors and produce it both ways. Let it cut M_1 in A and M_2 in B. An image of L will be formed at L_1 such that $L_1 A = LA$ (§111).

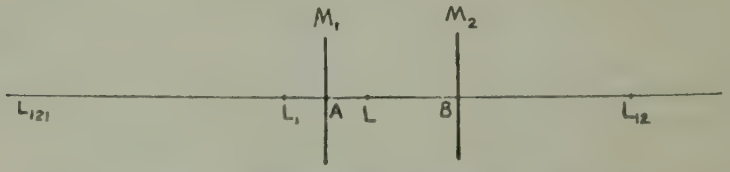

Fig. 68. Images formed by Parallel Mirrors.

L_1 is now in front of M_2. Therefore an image, L_{12}, will be formed such that $L_{12} B = L_1 B$.

L_{12} is now in front of M_1. Therefore an image, L_{121}, will be formed such that $L_{121} A = L_{12} A$, and so on.

Since an image is always in front of *one* of the mirrors, it is evident that an infinite series of images will be formed.

Another series, L_2, L_{21}, L_{212}, ... is formed by considering a first reflexion in M_2, but to avoid confusion these have been omitted in the diagram.

Path of Rays of Light. Suppose an eye is placed at E. By what path have the rays of light proceeding from L reached the eye, so as to enable it to see a given image, say L_{121} ?

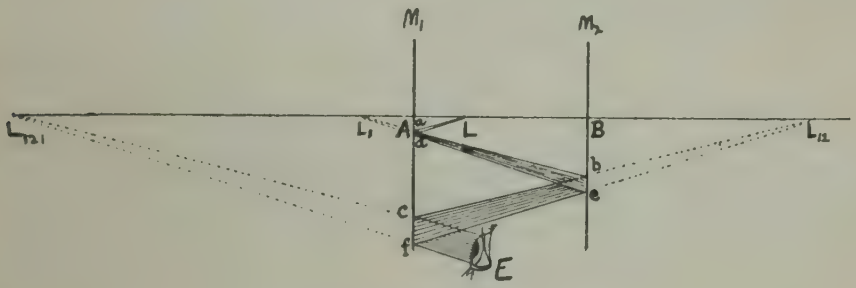

Fig. 69. Path of Rays of Light.

Join L_{121} to E as shown in fig. 69, the joining lines cutting M_1 in c and f. (N.B. If these joining lines do not cut the mirror, then an image formed at L_{121} is not visible to an eye placed at E).

Join c and f to L_{12}, cutting M_2 in b and e. Similarly join b and e to L_1, cutting M_1 in a and d, and lastly join a and d to L.

The eye sees the image L_{121} by light which has traversed the path L (ad) (be) (cf) E.

In this case the light has been reflected three times. At each reflexion there is some loss of light, and after a certain number of reflexions the light which remains is too faint to enable the corresponding image to be seen.

114. Mirrors Inclined at an Angle. Let AM_1 and AM_2 represent two mirrors inclined at an angle M_1AM_2, and let L represent an object placed between them.

L being in front of M_1, gives an image, L_1. Since L_1 is now in front of M_2, it gives an image, L_{12}, which in turn gives L_{121} and so on. The construction will be clear from the figure. This time, however, the number of images is not (even theoretically) infinite, for after a while an image is obtained which is behind *both* mirrors, and at this point (L_{121212} in the figure) the series comes to an end. The number of images in the series depends on the angle between the mirrors.

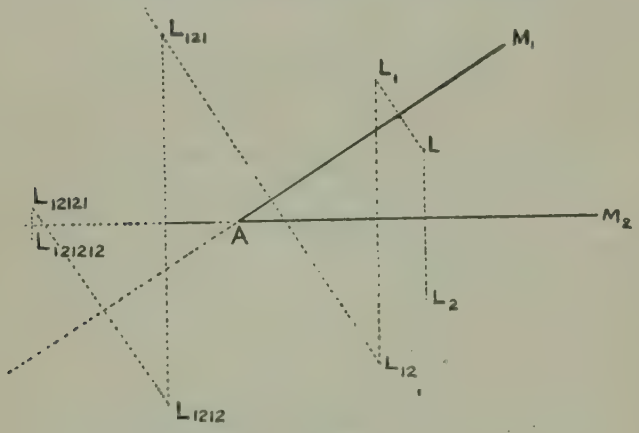

Fig. 70. Mirrors Inclined at an Angle.

There is, of course, another series of images starting with L_2 formed by reflexion in the mirror M_2, but it would confuse the figure to insert the other members of the series (L_{21}, L_{212}, etc.)

A rather important special case arises when the mirrors are at right angles.

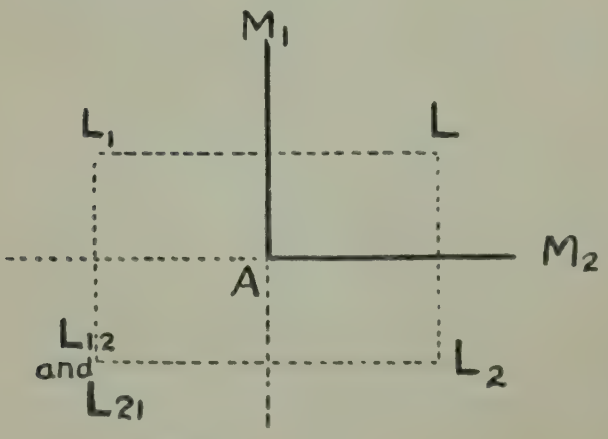

Fig. 71.

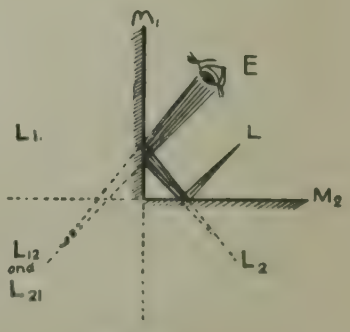

Fig. 72

It will be seen from fig. 71 that there are four images of which two (L_{21} and L_{12}) are coincident. Fig. 72 shows the path by which the reflected rays reach the eye.

QUESTIONS ON CHAPTER VI.

1. Give some reasons for believing that light travels in straight lines.—*Camb.* 1923.

2. Describe an experiment with full details by which it may be shown that light can travel in straight lines.

Mention two means by which light can reach an observer without coming along the straight line from the observer's eye to the source of light and explain what happens in each case.—*Oxf.* 1924.

3. A piece of white cardboard is stood upright on a table in a darkened room. Describe and explain the appearance of the card when, (1) a lighted candle is placed in front of it, (2) an opaque screen with a small hole is placed between the cardboard and the candle. In the latter case, what would be the effect of, (1) moving the cardboard farther from the perforated screen, (2) increasing the size of the hole in the screen ?—*Lond.* 1923.

4. Define the terms : Ray of light ; beam of light ; and shadow.

Explain with a diagram how an annular eclipse of the sun is caused.

Why are some eclipses of the sun annular and others total ? —*Oxf.* 1924.

5. Draw diagrams illustrating the formation of shadows by an opaque obstacle, and state what may be deduced therefrom concerning the propagation of light.

Draw, and explain fully, diagrams illustrating partial and total eclipse of the moon.—*C.W.B.* 1924.

6. How would you arrange to compare the powers of two small electric lights by means of a shadow photometer ; what measurements would you take ; how would the calculation be made ; and what is the main principle on which the method depends ?

Show by means of a diagram how the shadows would appear coloured if one of the lamps had red glass and the other blue. —*M.L.L.S.B.* 1925.

7. In what manner does intensity of illumination depend on the distance of a source of light? How do you justify your statement ?

A lamp is 8 ft. directly above one end of a 6 ft. table. How do the intensities of illumination at the two ends compare with one another?—*M.L.L.S.B.* 1926.

[N.B.—In this case one end of the table is receiving the light not at right angles, but at an angle whose cosine is 4/5, and it must be remembered that the intensity of illumination varies as the cosine of the angle of incidence. Thus the respective intensities are proportional to $1/8^2$ and $1/10^2 \times 4/5$. Cp. also No. 14 below.]

8. Describe the method of use of a simple type of photometer with which you are familiar.

Explain how you could determine by its aid the percentage of light cut off by placing a glass plate in front of a source.—*C.W.B.* 1921.

9. If I can see to read small print held 10 ft. from a 50 c.p lamp, how near must I hold it to a single candle? Explain the theory on which your calculation is based, and point out how the surroundings might affect the actual result.—*Lond.* 1926.

10. Describe and explain the appearance of a grease spot at the centre of a sheet of white paper when viewed, (a) by reflected light, (b) by transmitted light.

A 16 c.p. lamp is placed 50 cms. from such a sheet of paper. Where must a 25 c.p. lamp be placed in order that the grease spot may be indistinguishable?—*Lond.* 1921.

11. How does the intensity of illumination of a surface vary with its distance from the source of light? Being provided with a small source of light, a photometer, and some standard candles, describe how you would verify the truth of your statement.—*Lond.* 1922.

12. Describe the action and use of a simple photometer which you would employ to measure the candle-power of an electric lamp stated to be 25 c.p.

It may be assumed that a standard 16 c.p. lamp is available for the test.—*Lond.* 1925.

13. What is meant by the candle-power of a source of light? Describe and explain some method of comparing the candle power of two sources of light.

If a source of light and a 40 c.p. lamp produce the same illumination on a screen when placed 20 cms. and 40 cms. away respectively, how far from the source must a 100 c.p. lamp be placed in order to produce the same illumination as the source of light when placed 40 cms. away?—*Camb.*

14. A room is lighted by two lamps, each of 20 c.p., which are 8 ft. apart and 6 ft. above a table on which a book lies directly beneath one lamp, A. How does the illumination of the book, when both lamps are alight, compare with that given by A only?—*M.L.L.S.B.* 1926.

[See note to No. 7 above.]

15. What are the laws of reflexion of light? Describe an experiment by which they may be verified, pointing out carefully why it is that you consider that your observations verify the laws.—*Oxf.* 1924.

16. A plane mirror 2 ft. high is fixed on one wall of a room, the lower edge being 4 ft. 6 ins. from the ground. If the opposite wall of the room is 14 ft. distant and 10 ft. high, draw a diagram to show from what point a man must look in order to see reflected in the mirror the whole height of the opposite wall from floor to ceiling.—*Lond.* 1923.

17. Show how to arrange two plane looking-glasses so that a person by looking straight before him could get a side view of his own head.—*M.L.L.S.B.* 1922.

18. Explain the formation of images by a plane mirror. Show that a person will be able to see a full length image of himself in a mirror which is only about one-third his height.—*O.C.* 1921.

19. State the laws of reflexion of light.

A single plane mirror will deviate a beam of light through 180° only when the incidence is strictly normal, while two plane mirrors fixed at right angles will do this for various angles of incidence. Explain these facts, with diagrams.—*Camb.* 1925.

20. Show that when a plane mirror is turned through a given angle, a ray reflected from the mirror turns through twice that angle. Describe one practical application of this principle.—*Lond.* 1923.

21. A horizontal beam of light from a lamp falls on a mirror and is reflected at right angles horizontally, forming a spot of light on a screen 1 metre distant from the mirror. How must the mirror be placed? Through what angle must it be rotated to make the spot of light move 10 cms. on the screen horizontally? Justify the truth of the theorem which you employ in solving this problem.—*Lond.* 1926.

22. Two plane mirrors are placed back to back and are opened out gradually to form a wedge, the reflecting surfaces being on the outside. A beam of parallel light falls on the edge of the wedge, and it is found that the two beams reflected from its faces make ultimately an angle of 110° with each other. Find the angle between the mirrors, explaining carefully the principle of the method.—*C.W.B.* 1923.

23. What are the characteristics of the image of an object formed in a plane mirror? How would you verify each experimentally?—*Lond.* 1922.

24. Two plane mirrors are fixed parallel to one another on opposite walls of a room. Explain the formation of the series of images which are seen when an object is placed between them.

Draw a diagram showing the path of the rays by which an eye sees the third image in one of the mirrors.—*Lond*. 1921.

25. Two mirrors, A and B, are inclined to each other at 45°. An object, represented by the letter P, is placed with its lowest point ½ inch from A and 1 inch from B. Show in a diagram the position of the image of the P formed by two reflexions first at mirror A and then at mirror B. Draw also the path of the rays by which an eye at Q, 1½ inches from each of the mirrors, will see a point on this image.—*Lond*. 1924.

26. Two plane mirrors are placed at right angles to each other. Make a careful diagram to show, (1) the images formed by the mirrors of an object placed between them, (2) how the light travels from the last image formed to the eye of an observer also situated between the mirrors.—*Camb*. 1923.

Chapter VII.

REFLEXION AT CURVED SURFACES.

115. We shall begin with definitions of a few terms that are being constantly used.

Spherical Mirror. *A spherical mirror is a reflecting surface consisting of a portion of a sphere.*

Such a mirror may be *concave* or *convex*.

Centre of Curvature. The *centre of curvature* of a concave or convex spherical mirror is the centre of the sphere of which the mirror forms a part.

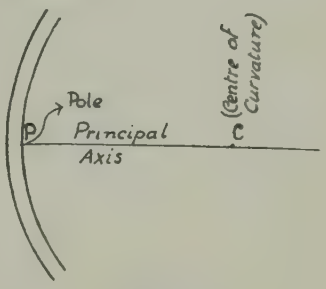

Fig. 73.

Pole. The *pole* of a spherical mirror is the central point of its surface.

Principal Axis. The *principal axis* is the straight line joining the centre of curvature to the pole.

116. Principal Focus. Consider a ray of light, A B, travelling parallel to the principal axis of a concave mirror, and striking the mirror at B. Join BC (C being the centre of curvature).

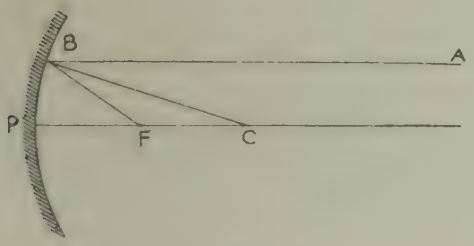

Fig. 74.

Now although the surface of the mirror is curved, a very small portion of it in the neighbourhood of any given point, such as B, may be regarded as plane. BC is the normal at B. Hence AB will be reflected along BF, where $\angle CBF = \angle CBA$. Let the reflected ray cut the principal axis in F.

By construction, $\angle CBF = \angle CBA$.

Also by parallels, $\angle BCF = \angle CBA$ (alternate angles).

$\therefore \angle CBF = \angle BCF$.

$\therefore FC = FB$.

Now if the incident ray is near the principal axis, B will be close to P, and then FB = FP (approximately).

$\therefore$ FC = FP (approximately).

Evidently the same reasoning would apply to any ray parallel to (and near to) the principal axis. Hence we may say that *all rays incident on a concave spherical mirror in a direction parallel to the principal axis, and travelling near to that axis, will after reflexion pass through a point mid-way between the pole of the mirror and its centre of curvature.* This point is known as the **principal focus** of the mirror.

The distance PF (i.e., from the pole of the mirror to the principal focus) is known as the **focal length** of the mirror.

117. Image of a Point. Suppose A is a luminous point at some distance from a concave mirror. It gives off rays in all directions, and of these the one travelling along AC, i.e., along a normal, will be reflected along its own path, while the ray AB will be reflected along BF.

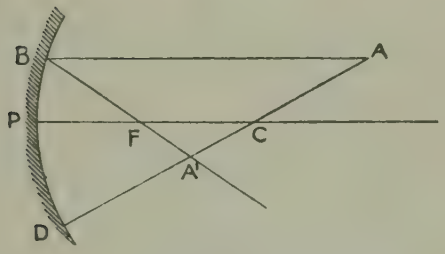

Fig. 75.

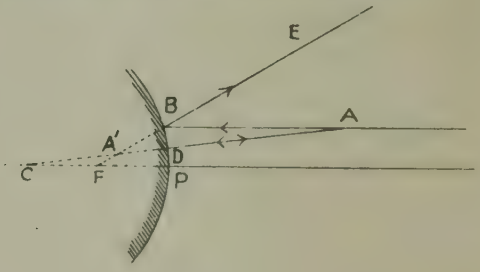

Fig. 76.

These two rays intersect in A′, and it can be proved that *all* the reflected rays will pass through (or very near) A′.

Hence A′ is the image of A.

Now consider the case of a *convex* mirror.

AB will be reflected along BE as though the reflected ray had passed through F; and AD (normal) will be reflected along its own path. Hence the rays which actually started from A will seem to the eye to proceed from A′, which is therefore the image of A.

118. Real and Virtual Images. There is evidently a great difference between the two images obtained in the last paragraph. In the first one (concave mirror) A′ is seen by rays of light *actually proceeding from the point.* Such an image is said to be **real.** In the second (convex mirror), A′ is seen by rays of light *which do not actually proceed from the point, but only appear to do so,* and such an image is a **virtual** one.

[Q.—Is the image obtained by a plane mirror, as in §111, rea or virtual ?]

119. Conjugate Foci. In fig. 75 we saw that rays of light from A are brought to a focus at A′. If, however, A′ had been the source of light, it is easy to see that the rays of light proceeding from it would, after reflexion, have been brought to a focus at A. A and A′ are said to be ***conjugate foci,*** which we may define as being **two points, such that if a very small source of light is placed at one of them, an image is formed at the other, and vice versa.**

120. Graphical Solutions. Many problems on mirrors may be solved graphically. In working them we should always keep in mind that :—

(a) A ray of light striking the mirror normally will be reflected along its own path.

(b) A ray parallel to the principal axis will be reflected through the principal focus.

(c) A ray passing through the principal focus will be reflected parallel to the principal axis.

Example 32. A concave mirror has a radius of curvature of 3 inches, and an object $\frac{1}{2}$ inch in height is placed, (a) $3\frac{3}{4}$ inches, (b) 1 inch, (c) $1\frac{1}{2}$ inches from it. Find graphically in each case the position and size of the image.

Case (a). AB represents the object. The method of obtaining A′ (the image of A) will be clear from the figure (ray of light parallel to principal axis reflected through principal focus, and vice versa).

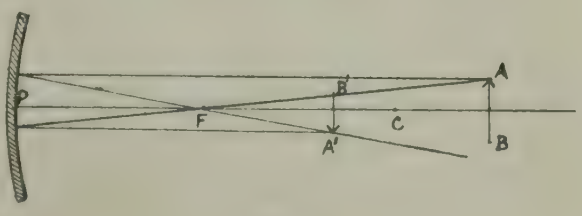

Fig. 77

To avoid confusion, the lines which determine the position of B′ have been omitted.

A′B′ is found by measurement to be **2½ inches from the mirror** and to be **0.33 inches high.**

Note that the image is *real* and *inverted*.

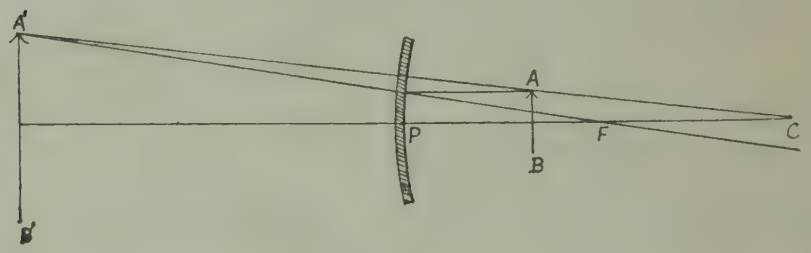

Fig. 78

Case (*b*). As before, only the rays from A have been actually drawn. The image this time is *virtual* and *erect*. It is found to be **3 inches behind the mirror** and to be **1½ inches high.**

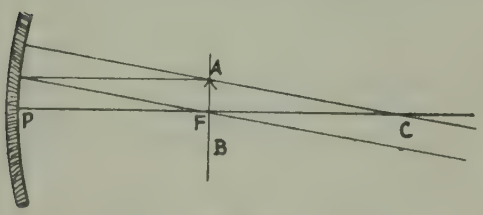

Fig. 79.

Case (*c*). The drawing shows that after reflexion the rays from A are parallel. Hence no image of A is formed at all (or, in the language of Geometry, A gives an image at infinity). Similarly of course for B.

The student may work out graphically the case of a small object placed in front of a *convex* mirror (he will obtain some help from fig. 76). In this case, whatever the position of the object, the image is always found to be *erect, virtual* and *reduced in size.*

121. An Important Formula. Suppose an object, AB, in front of a spherical mirror gives an image, A′ B′.

Let u = distance of object from mirror.

v = distance of image from mirror.

f = focal length.

Then it can be proved that $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

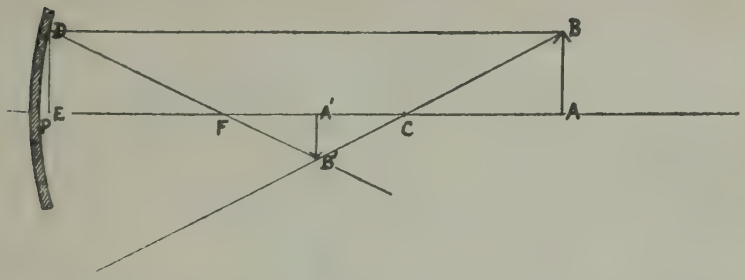

Fig. 80.

(The proof is omitted, but it might easily be worked out by a student familiar with the properties of similar triangles. Fig. 80 gives all the necessary construction.)

122. Magnification. Magnification is defined as **the length of the image divided by that of the object.**

Thus in fig. 80, magnification $= \dfrac{-A'B'}{AB} = -\dfrac{A'B'}{AB}$

(*Minus*, because if AB, measured above the axis, is regarded as positive, A'B', measured below it, must be negative.)

But $A'B'/AB = A'B'/ED = FA'/EF$ (by similar triangles) $= \dfrac{v-f}{f}$

Also from $1/v + 1/u = 1/f$ we obtain $f = \dfrac{uv}{u+v}$

$$\therefore \text{ magnification } = -\frac{A'B'}{AB} = -\frac{v-f}{f} = -\frac{v-\dfrac{uv}{u+v}}{\dfrac{uv}{u+v}}$$

Which reduces to $-v/u$.

Hence we see that (*numerically*)

$$\textbf{Magnification} = \frac{\textbf{length of image}}{\textbf{length of object}} = \frac{\textbf{distance of image}}{\textbf{distance of object}}$$

With regard to sign, note that if the magnification is *negative*, the rays (fig. 80) must have crossed at C, and therefore the image is *inverted*. If the magnification is positive, the image is erect.

123. Convention as to Signs. In all problems relating to mirrors (and lenses) the following conventions as to signs must be carefully observed.

(i.) Always measure *from* the mirror (or lens).
(ii.) If the measurement carried out as in (i.) is *opposite* to the direction of the incident light, the sign is *plus*; otherwise, *minus*.

Example 33. For this we will do by calculation Example 32 (pp. 115, 116), previously done graphically.
(a) Since $r = 3$ inches, $\therefore f = +1\frac{1}{2}$ inches.
(Plus sign, because in measuring from the mirror to the principal focus, we are going opposite to the direction of the light.)
Also distance of object $(u) = 3\frac{3}{4}$.
Substituting in formula $1/v + 1/u = 1/f$,
We have $1/v + 1/3\frac{3}{4} = 1/1\frac{1}{2}$.
Solving this equation, we obtain $v = +2\frac{1}{2}$ inches, i.e., the image is formed **$2\frac{1}{2}$ inches in front of the mirror.**
To obtain the size of the image, we have (if I = length of image and O = length of object)
$I/O = -v/u$.
$\therefore I/\frac{1}{2} = -2\frac{1}{2}/3\frac{3}{4}$.
This gives **$I = -\frac{1}{3}$ inch** (minus sign indicating that image is **inverted**).
(b) Here $f = +1\frac{1}{2}$ as before, $u = +1$.
Substituting these values in the formula $1/v + 1/u = 1/f$,
$\therefore$ we obtain $v = -3$ inches.

As we must always measure from the mirror, and the sign is minus, it is evident that as we proceed from the mirror to the image we are going in the same direction as the incident light.
$\therefore$ the image is **3 inches behind the mirror.**
Again, magnification $I/O = -v/u$.

$$\therefore \frac{I}{\frac{1}{2}} = -\frac{-3}{1} = 3$$

$\therefore I = +1\frac{1}{2}$, so that the image is **erect**, and **$1\frac{1}{2}$ inches high.**
(c) $f = +1\frac{1}{2}$ and $u = +1\frac{1}{2}$.
Substituting in formula $1/v + 1/u = 1/f$,
We have $1/v + 1/1\frac{1}{2} = 1/1\frac{1}{2}$.
$\therefore 1/v = 0$
$\therefore v = \infty$, agreeing with the result found in §120.

Example 34. An object 1 inch in height is placed 3 inches in front of a convex mirror whose radius of curvature is 4 inches. Find the position, size and nature of the image ("nature," i e., whether real or virtual, and whether erect or inverted).

Measuring from the mirror to the centre of curvature, we are evidently going in the same direction as the incident light.
$\therefore r = -4$ and $f\ (= r/2) = -2$. $u = +3$.
$1/v + 1/u = 1/f$.
$\therefore 1/v + 1/3 = 1/-2$.

Solving this, we obtain $v = -6/5 = -1\frac{1}{5}$.

Magnification $= -\frac{v}{u} = -\frac{-6/5}{+3} = \frac{2}{5}$. (∴ image is two-fifths the length of the object).

These results show that the image is formed $1\frac{1}{5}$ inches *behind* the mirror (hence it is virtual). It is $\frac{2}{5}$ inches high, and erect (because magnification was found to be positive).

Example 35. Where must an object be placed in front of a concave mirror of focal length 20 cms., in order that the image may be magnified threefold, (a) when the image is real, (b) when it is virtual?—*Lond.* 1921.

In case (a) image is real,
∴ it is in front of the mirror.
Hence measuring from the mirror to the image, we are going against the incident light.
∴ v is $+^{ve}$. Similarly u and f are both $+^{ve}$.
∴ magnification $(= -v/u)$ is $-^{ve} = -3$. (∴ image is inverted).
Since $-v/u = -3$, we have $v = 3u$.
∴ from formula $1/v + 1/u = 1/f$,
We have $1/3u + 1/u = 1/20$.
Solving this, we obtain $u = \mathbf{26\frac{2}{3}}$ **cm.**

In case (b), image is virtual,
∴ v is $-^{ve}$, while u and f are $+^{ve}$ as before.
∴ magnification $(= -v/u)$ is $+^{ve} = 3$ (∴ image is erect).
Since $-v/u = 3$, we have $v = -3u$.
∴ $1/v + 1/u = 1/f$
becomes $\frac{1}{-3u} + \frac{1}{u} = \frac{1}{20}$
whence $u = \mathbf{13\frac{1}{3}}$ **cm.**

Example 36. When a pin is held 20 cms. in front of a concave mirror, an erect image is formed at a distance of 60 cms. from the pin. Calculate the focal length of the mirror and verify the calculation by a diagram drawn to the scale of 1 inch = 10 cms. —*Lond.* 1922.

Image is erect.
∴ magnification is $+^{ve}$
i.e., $-v/u$ is $+^{ve}$
∴ v and u are of opposite sign, and as u is $+^{ve}$, v must be $-^{ve}$,
i.e., image is formed *behind* mirror.
Image is 60 cms. from object,
∴ it is 40 cms. from mirror.
Hence $u = 20$, $v = -40$, $f = ?$
Substituting in $1/v + 1/u = 1/f$, we have $-1/40 + 1/20 = 1/f$.
∴ $f = 40$.

In working out a graphical solution it is convenient to assume the answer ($f = 40$ ∴ $r = 80$) and see if under these circumstances

the image obtained is erect, and 60 cms. from the pin. This will be found to be the case. The drawing is left as an exercise to the student. It is very similar to Example 32, *Case* (*b*).

124. Relation of Object and Image. Given u and f, we can calculate v, as seen in the preceding examples. We can then find :—

(i.) Relative *size* of image, for magnification $= -v/u$.

(ii.) Whether image is *erect or inverted* (erect if magnification is $+^{ve}$ and vice versa).

(iii.) Whether image is *real or virtnal* (real if v is $+^{ve}$ virtual if it is $-^{ve}$).

125. How v changes with u. For clearness let us take a particular case, $f = 8$ say. Then in the formula, $1/v + 1/u = 1/f$, we can give any values we like to u, obtain corresponding values of v, and express the results graphically, thus :—

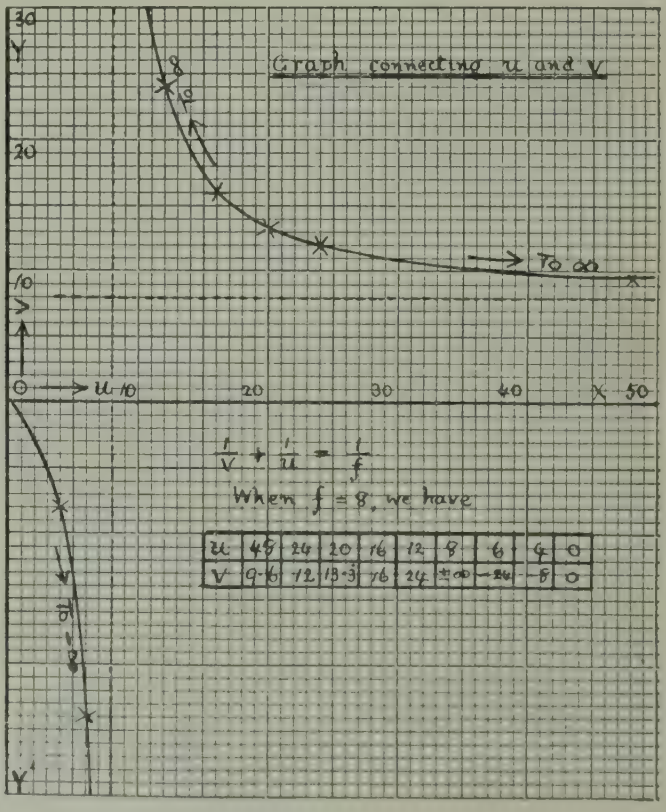

Fig. 81. Relation between u and v (Concave Mirror).

Experimental Work.

126. To Find Radius of Curvature of Concave Mirror. We may find this by:—

(i.) A geometrical method.

(ii.) A direct optical method.

(iii.) An indirect optical method, in which we assume the formula, $1/v + 1/u = 1/f$.

(i.) *Geometrical Method.* To obtain a really accurate result the spherometer should be employed.

An approximate result, however, may be obtained as follows: A piece of rather stout lead wire is pressed against the concave surface of the mirror so as to bisect it. On removing the wire gently (so that it shall retain its shape) we have an arc which may easily be copied on paper (ABC, fig. 82a). By a familiar construction, indicated in the figure, we can easily find O, the radius of the corresponding circle, and OA is then the required radius of curvature.

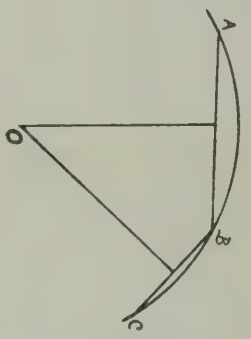

Fig. 82a.

(ii.) *Direct Optical Method.* This depends on the principle that if a luminous point is placed at the centre of curvature of a concave mirror, the rays from it will all

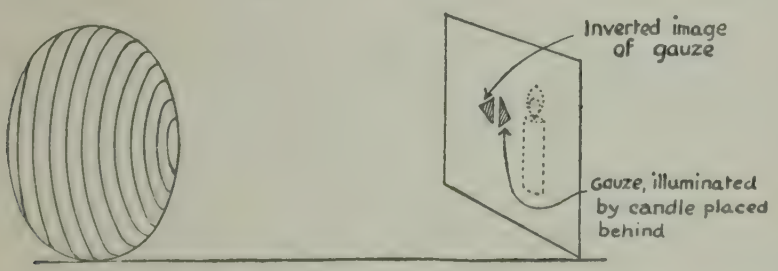

Fig. 82b. Finding Radius of Curvature.

be reflected along their original paths, and therefore the image formed will coincide with the object. The distance of the object from the mirror now gives the radius of curvature.

In practice the luminous object often consists of a small triangle of wire gauze mounted on cardboard, and illuminated by a candle placed just behind it. A sharply defined image may be obtained practically coinciding with the gauze. The experiment must be carried out in a darkened room.

(iii.) *Indirect Optical Method.* A straight line, ABC, is ruled along the bench. At A is placed the mirror, at B a screen, and at C a lighted candle. The screen is placed slightly to left or right of the line, so that an image of the candle flame, produced by reflexion from the mirror, may be formed upon it.

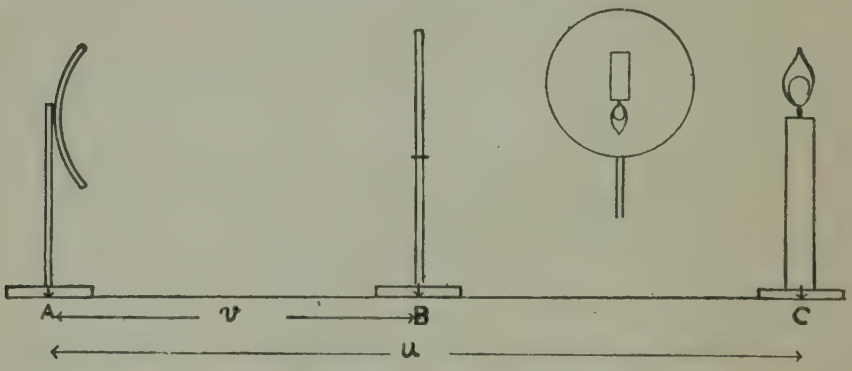

Fig. 83. Finding Radius of Curvature.

The distances u and v (indicated in fig. 83) are measured, and f calculated from the formula $1/v + 1/u = 1/f$; we then obtain r $(=2f)$.

Greater exactness may be secured by using illuminated gauze instead of the candle.

An advantage of the method just described (using either candle or gauze), is that a whole series of corresponding values of u and v may be found, and corresponding values of f calculated. The mean of these values gives a result of a considerable degree of accuracy.

(iii.a) *Distant Source of Light.* A simple but important special case of the last method occurs when the source of light is at a great distance. For if $u = \infty$, we have $1/u = 0$, and the formula then becomes $1/v + 0 = 1/f$, whence $f = v$.

Thus we may arrange the mirror so that it faces the sun, and so that an image of the sun is received on a screen. The distance from the mirror to the screen is then clearly equal to f.

In practice, the window frames a few yards from the mirror are often used as the distant object.

127. Verification of Formula. In §126 (iii.) we assumed the truth of the formula $1/v + 1/u = 1/f$, and used it to find f. To *verify* the formula, we must find f by some other method, say that of §126 (i.), and find if the value thereby obtained agrees with that found by the formula.

128. Focal Length of Convex Mirror.

(1). We may use the geometrical method of §126 (i.)

(2.) We may use the formula $1/v + 1/u = 1/f$.

The difficulty is that the image obtained by a convex mirror is always virtual, and therefore not very easy to "place."

However, by scratching off a little of the silvering we may obtain a direct view, at one and the same time, of (a) a pin mounted vertically on a block behind the mirror, (b) the virtual image of a candle flame, the lighted candle having been placed in front of the mirror.

We move the mounted pin forwards or backwards until it coincides with the image.

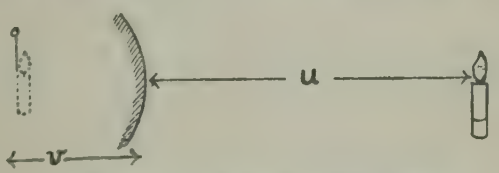

Fig. 84. Focal Length of Convex Mirror.

Note that in this case v is *negative*, because as we measure from the mirror to the image we are going in the same direction as the oncoming light. f will also work out to a negative quantity.

QUESTIONS ON CHAPTER VII.

1. Assuming the laws of reflexion, show why a parallel beam of light, after falling directly on a convex mirror, is converted into a diverging pencil of rays.

Why do images seen in a convex mirror always appear diminished in size?—*M. L.L.S.B.* 1926.

2. What do you understand by the terms: Pole, focal length, centre of curvature?

Prove that when an object is at a distance equal to two-thirds of the focal length from a concave mirror, the image produced is erect and virtual, and magnified three times.—*Camb.* 1925.

3. A concave mirror has a focal length of 20 cms. Where must an object be placed so as to form an image twice the size when the image is (a) real, (b) virtual? Draw clear diagrams in illustration.—*Lond.* 1923.

4. Prove the relation between the focal length and the radius of curvature of a concave mirror.

Where would you place a concave mirror of focal length 1 ft., so as to throw an image of a candle flame on a wall distant 10 ft. from the candle?—*Lond.* 1924.

5. A concave mirror has a radius of 10 ins. In what positions of the object will the image formed have a magnification of 3 diameters? Show by a diagram the paths of the rays forming the image in each case.—*Camb.* 1921.

6. Give an account of a method of determining the foca length of a concave mirror.

An object placed 10 cms. in front of a concave mirror is found to produce an image of half its size. Calculate the focal length of the mirror and the distance between the image and the mirror. —*Camb.* 1922.

7. How would you employ a concave spherical reflector and a point source of light in order to make, (a) a parallel beam, and (b) a divergent beam of light?

If the radius of curvature of the mirror was 2 ft., and the light was placed 1 ft. 4 ins. away from it, to what point would the beam converge after reflexion?—*M.L.L.S.B.* 1926.

8. Explain the formation of real and virtual images by a concave mirror.

A candle is placed 9 ft. from a wall, and a concave mirror is held so that an image magnified 10 times is thrown on the wall. Find the position of the mirror and its focal length.—*C.W.B.* 1922.

9. Distinguish between a real and a virtual image.

How far from a concave mirror of radius 3 ft. would you place an object to give an image magnified 3 times? Would the image be real or virtual?—*O.C.* 1924.

10. You are given a piece of mirror glass which may be plane or slightly curved. What experiments would you make to determine whether it is plane, convex or concave? Illustrate your answer by diagrams.—*Camb.* 1923.

11. How does the size of an image formed by a concave mirror vary as an object is brought from a great distance up to the focus of the mirror. Give illustrative diagrams.—*Camb.* 1923.

12. A man walks from a distance towards a large spherical concave mirror, some feet in diameter, which is mounted on a wall opposite to him. With the aid of a series of diagrams, show and explain the changes in the appearance of the image of himself which he sees.—*M.L.L.S.B.* 1925.

13. Describe a method of determining the radius of curvature of a concave mirror.

The radius of curvature of a concave mirror is 40 cms. Find where the object must be placed so that the image is virtual and 3 times the size of the object.—*O.C.* 1925.

14. (a) What do you understand by the expression, "conjugate foci of a spherical mirror"?

(b) Describe a method for finding the focal length of a concave mirror by measuring the distance from the vertex of the mirror of a pair of conjugate foci situated on its principal axis. Mention any practical difficulties and say how they are surmounted.

(c) If these distances were 5 ins. and 10 ins., calculate the focal lengths that the mirror might have.—*Oxf.* 1923.

15. Describe a method of determining the focal length of a concave mirror.

The radius of curvature of a concave mirror is 20 cms. Find where an object must be placed so that the image is twice the size of the object and, (a) real, (b) virtual.—*Camb.* 1924.

16. What is the "focal length" of a concave mirror? How would you measure it experimentally?

Such a mirror gives a 3-times magnified image of an object placed 20 cms. from it. Find the focal length if the image is, (a) erect, (b) inverted.—*C.W.B.* 1925.

17. State the laws of reflexion of light. Show why a concave mirror produces an image of an object placed on its axis.

Under what circumstances is this image, (a) real, (b) virtual, (c) erect, (d) inverted, (e) enlarged, (f) diminished?—*C.W.B.* 1926.

Chapter VIII.

REFRACTION AT PLANE SURFACES.

129. Meaning of Refraction. So long as it is confined to the same homogeneous medium, light travels in straight lines (§97). If, however, after travelling in one medium (say air) it enters another (say glass), there is a change of direction at the surface of separation of the two media.

***Refraction* is the change in direction which a ray of light undergoes when it passes from one medium to another.**

130. Sine of an Angle. In what follows there will from time to time be references to the *sine* of an angle, and for the benefit of students who have done no trigonometry this term will be explained now.

Let POM be any angle, and from P, any point on one of the arms of the angle, let PM be drawn perpendicular to the other arm. The side PM, opposite to the angle (POM) under consideration, we shall call the *perpendicular*. OP is, of course, the *hypotenuse*. Then the **sine** of the angle POM (written sin POM) is the ratio $\frac{\textbf{perpendicular}}{\textbf{hypotenuse}} = \frac{PM}{OP}$

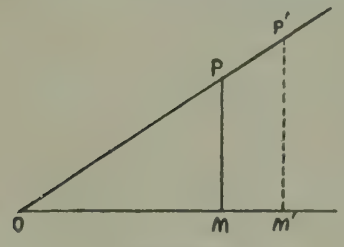

Fig. 85.

Note that, for the same angle, the sine is constant for all positions of P. Thus if we take P′ instead of P (fig. 85), we obtain a longer perpendicular P′M′, but we obtain a proportionally longer hypotenuse OP′, and the ratio P′M′/OP′ is equal to the ratio PM/OP.

131. Laws of Refraction.

1. **The refracted ray lies in the plane containing the incident ray and the normal.**

2. **For a given pair of media, and for light of a given wave-length, a constant number is obtained on dividing the sine of the angle of incidence by the sine of the angle of refraction.**

The reference to light of a given wave-length will be understood after reading §153.

Suppose light is passing from a medium, A, to another one, B, the angle of incidence at the surface of separation being a and the angle of refraction b. Then the constant ratio, $\sin a / \sin b$, is known as the *index of refraction* from A to B, and is written as ${}_A\mu_B$

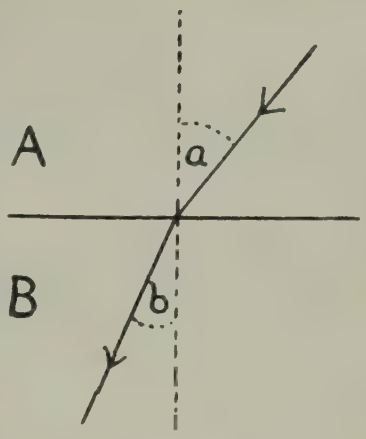

Fig. 86.

If the first medium is air, the constant ratio is spoken of simply as the *index of refraction* or *refractive index* of the second medium, and is written as μ (pronounced *mu*).

132. Experimental Proof. PQRS (fig. 87) represents in plan a thick slab of glass laid face downwards on a sheet of paper. A and B represent two pins placed behind the glass.

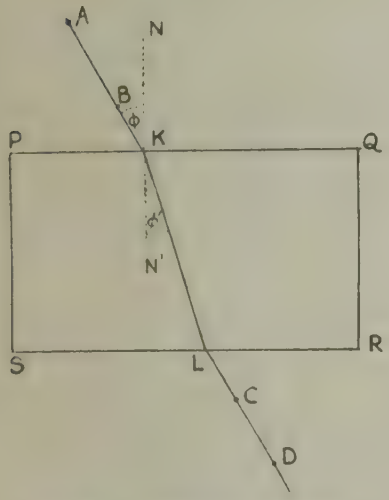

Fig. 87. Laws of Refraction.

Looking through the glass from the front, we place two other pins, C and D, so that A, B, C and D appear to be in a straight line.

Now with a pencil, mark the outline of the slab of glass, and also the position of the pins.

Evidently a ray of light which entered the glass by the path ABK must have left it by the path LCD, and since between K and L the light was travelling in the same medium (glass) we may assume that its path

between K and L was a straight line. Hence we have been able to trace completely the path of the ray.

At K draw the normal, NKN′. The angle of incidence is AKN ($=\phi$) and the angle of refraction is LKN′ ($=\phi'$). From tables look out sin ϕ and sin ϕ', and work out the ratio $\frac{\sin\ \phi}{\sin\ \phi'}$.

Repeat the experiment for various angles of incidence, obtaining corresponding angles of refraction. It will be found that the ratio $\frac{\sin\ \phi}{\sin\ \phi'}$ is constant. This constant ratio will be the refractive index of glass.

With regard to the first law, we may apply the same argument as was used in connection with the corresponding Law of Reflexion (§108).

Generally speaking, it is found that when a ray of light passes from a rarer medium into a denser one, it is bent *towards* the normal, and vice versa.

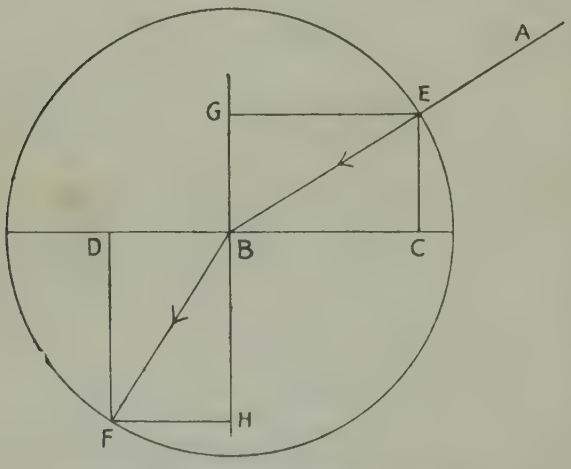

Fig. 88. Construction for Refracted ray.

133. Graphical Construction for Refracted Ray. Suppose a ray of light in one medium (e.g., air) is travelling into another (e.g., glass). We require to find the new direction. Suppose the index of refraction is given as 1.5, and that the incident ray AB strikes the surface of separation at B. Mark off BD = 1 unit, and BC = 1.5 units, as indicated in the figure. Draw CE perpendicular to BC, cutting AB in E. With centre B and radius BE, describe a circle.

From D draw DF perpendicular to DB to cut this circle in F. Join BF. Then shall BF be the required refracted ray.

To prove it, draw GBH perpendicular to BC, and draw EG and FH perpendicular to GBH.

$$\text{Then } \frac{\sin EBG}{\sin FBH} = \frac{EG/EB}{FH/FB} = \frac{EG}{FH} \text{ (for } EB = FB) = \frac{BC}{BD} = \frac{1.5}{1}$$

But EBG is the angle of incidence.
$\therefore$ FBH is the angle of refraction,
and hence BF is the refracted ray.

In the case just considered, the ray of light was travelling from the rarer medium to the denser one, and was refracted towards the normal. If the upper medium had been the denser one, we should have had to make BC = 1 unit and BD = 1.5 units. The student should make for himself a careful drawing of the figure in this case.

If the use of a table of sines is allowed, the path of the ray may be traced out a little more quickly and accurately by trigonometry. Thus in fig. 88 suppose $\angle EBG = 50°$,

$$\text{Now } \frac{\sin EBG}{\sin FBH} = 1.5,$$

And $\sin EBG = \sin 50° = .766$.

By substitution we find that $\sin FBH = .511$, and by reference to a table of sines we find that $\angle FBH = 30.7°$.

134. Apparent Depth of Water. Suppose A is a point at the bottom of a layer of water, and of the rays of light proceeding from A, let us specially consider two, (i.) one travelling along the normal, AN, and (ii.) another one, AB, very near to the normal. AN travels into the air along NC

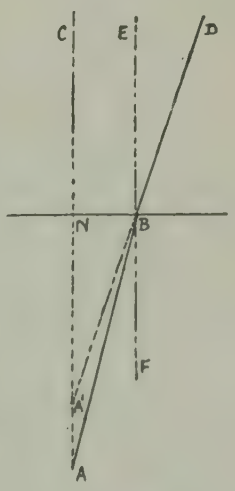

Fig. 89.
Real and Apparent Depth.

without change of direction, while AB is refracted along BD. To an eye placed so as to receive these rays, A will appear to be at A′, the point where CN and DB meet if produced.

At B draw EBF perpendicular to NB. Now if μ = index of refraction from air to water (actually about $\frac{4}{3}$), $1/\mu$ = index from water to air.

$$\therefore \frac{\sin ABF}{\sin EBD} = \frac{1}{\mu}$$

But by parallels, $\angle ABF = \angle BAN$ and $\angle EBD = \angle NA'B$.

$$\therefore \frac{\sin BAN}{\sin NA'B} = \frac{1}{\mu}$$

$$\therefore \frac{BN/AB}{BN/A'B} = \frac{1}{\mu}$$

$$\therefore \frac{A'B}{AB} = \frac{1}{\mu} \text{ or } \frac{AB}{A'B} = \mu$$

Now as AB is supposed to be very close to AN,

$$\frac{AB}{A'B} = \frac{AN}{A'N} \text{ (approximately)} = \frac{\text{real depth}}{\text{apparent depth}}$$

Hence we have the important relation $\dfrac{\textbf{real depth}}{\textbf{apparent depth}} = \mu$.

As μ for water is about $\frac{4}{3}$, we see that the real depth is about a third more than the apparent depth.

In the case just considered, one of the rays (ANC) is normal, and the other (ABD) is nearly so. If instead of ANC and ABD, we had taken two more oblique rays, we should have found that, after refraction, the point from which they appeared to diverge was nearer the surface than A′.

Thus the apparent depth of water looked at in an oblique direction, is less than that observed when we look straight down into it.

135. A Method of Finding μ. The above result gives us a means of finding μ for water (or other transparent medium).

Let A be a small object at the bottom. The problem is to find A′, the apparent position of A. An object such as a knitting needle, KL, is clamped horizontally in a stand, and its image in the liquid is observed. The needle is raised or lowered, until, by the method of parallax, we find that its image, K′L′, coincides with A′.

Now NA′ = NF (§111).

Hence by measuring NF we have the apparent depth, and dividing this into the real depth, we at once obtain μ.

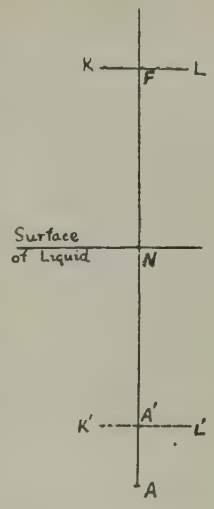

Fig. 90.
Finding Apparent Depth

136. Total Reflexion. When a ray of light travels from a rarer to a denser medium, it is in general refracted *towards* the normal. The angle of refraction is thus less than the angle of incidence, and for every angle of incidence there will be a corresponding angle of refraction.

But when the ray of light is travelling from the denser to the rarer medium, it is refracted *away from* the normal. The greater the angle of incidence, the greater becomes the angle of refraction. Thus in the figure A_1O is refracted along OA_2, B_1O along OB_2, and C_1O along OC_2. What happens if we make the angle of incidence still greater, for the angle of refraction is now as great as it can be (i.e., 90°)? Experiment shows that in such a case the incident ray is not refracted at all, but is

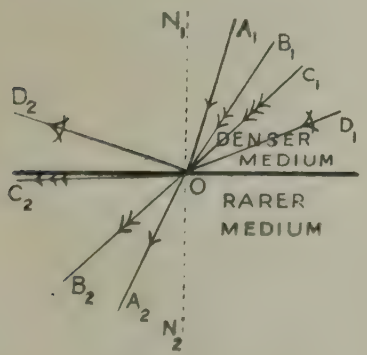

Fig. 91.

totally reflected at the surface of the medium. Thus the ray D_1O is reflected along OD_2 according to the usual law, the angle N_1OD_2 being equal to N_1OD_1. That angle of incidence for which the angle of refraction is 90° is known as the **critical angle.**

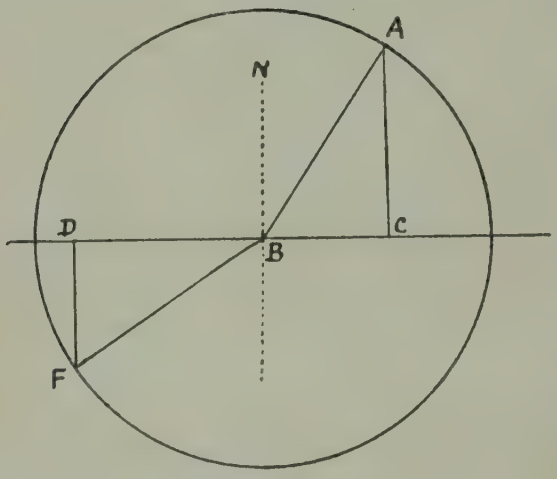

Fig. 92.

To understand the situation more exactly, let us apply the graphical construction of §133, taking $\mu = 1.5$ as before. AB now represents the ray in the denser medium. We measure off BC = 1 unit and BD = 1.5 units. From C we draw CA perpendicular to BD to meet AB at A. Then we draw a circle with centre B and radius BA, and finish as indicated in the figure, BF being the path of the ray in the rarer medium.

This construction will evidently be possible (i.e., for every incident ray there will be a corresponding refracted ray) until for a certain angle of incidence the radius becomes equal to BD.

Now BD/BC = .

$\therefore$ when radius = BD, we have $AB/BC = \mu$.

$\therefore$ $BC/AB = 1/\mu$,

i.e., $\sin BAC = 1/\mu$.

But BAC = NBA (alternate angles) = angle of incidence.

$\therefore$ *total reflexion occurs when the angle of incidence attains a value* ϕ *such that* $\sin \phi = 1/\mu$.

137. Graphical Construction for Critical Angle. This readily follows from the preceding discussion. Draw BD of μ units of length, and BC of 1 unit. With B as

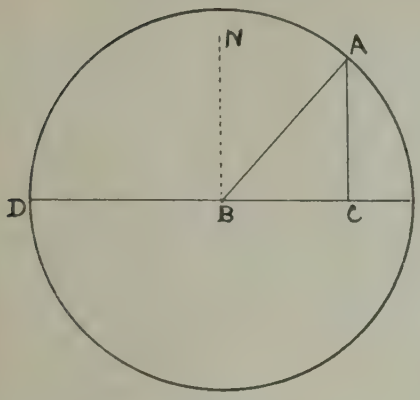

Fig. 93.
Construction for Critical Angle.

centre and BD as radius, describe a circle. Draw CA at right angles to DC to cut this circle in A. Join AB. Then AB represents a ray incident at the critical angle.

For sin NBA = sin BAC (parallels) = BC/AB = BC/BD = $1/\mu$.

$\therefore$ NBA is the critical angle.

138. Critical Angle Phenomena. Many every-day phenomena may be explained by the principles already discussed in connection with total reflexion and critical angle. Thus :—

(1.) If a tumbler partly filled with water, and containing a spoon, is held above one's head and a little to one side, the surface of the water appears as a mirror in which the lower part of the spoon is clearly reflected, the upper part of the spoon being invisible. Rays of light from below, including those from the lower part of the spoon, strike the surface at an angle greater than the critical angle, and are therefore totally reflected. Similarly,

(2.) A crack in a beaker, pane of glass, etc., which may be invisible from the front, appears as a brilliant streak when viewed from one side.

(3.) Pebbles, etc., at the bottom of a shallow pond and a few yards from the shore, are invisible, in spite of the short distance and the transparency of the water.

139. Total Reflexion Prism. The index of refraction (μ) for crown glass is 1.52. This gives $1/\mu = .666$, and the critical angle for crown glass is therefore $41\frac{3}{4}°$, because sin $41\frac{3}{4}° = .666$. Suppose a ray of light strikes a prism (of angles 90°, 45° and 45°) placed as shown in the

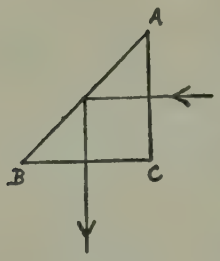

Fig. 94.

figure. It is incident on AB at an angle of 45° (i.e., greater than the critical angle), and so it is totally reflected as shown. Such a prism may therefore be used as a mirror.

[Q.—Need the angles be 90°, 45° and 45° ? Would such a mirror possess any advantage over the usual silvered variety ?]

140. Refraction through Plate. Let PQRS be a plate of glass or other transparent medium with parallel sides, PQ, RS, and let AB represent a ray of light incident on PQ at an angle ϕ. It will be refracted at an angle ϕ' given by $\frac{\sin \phi}{\sin \phi'} = \mu$. The ray will strike SR at an angle ϕ' and will emerge at an angle ϕ. From this it can easily be seen that its final direction is the same as its original one, though CD is not in the same straight line as AB. We express this by saying that CD has been *displaced* but not *deviated*, deviation being the angle between the final direction of a ray of light and its original direction (in this case, of course, 0°).

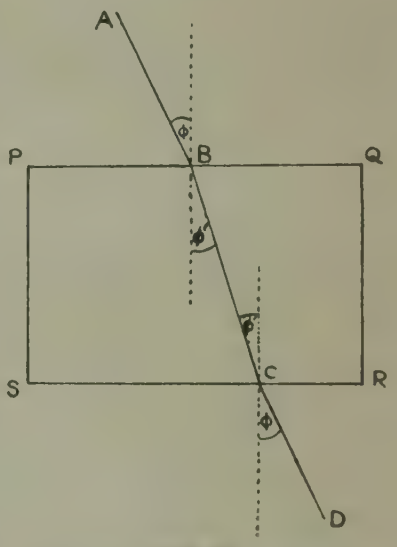

Fig. 95.
Displacement without Deviation.

141. Refraction through Prism. Let AB be a ray of light striking PR, the surface of a prism of given refractive index (for glass, $\mu = 1.5$). Applying the construction of §133, we can determine BC, the path of the ray within the prism, and similarly we can determine CD, the direction in which it emerges. The two faces, at which the ray enters and emerges respectively, of course meet along

an edge, and this is known as the *refracting edge* of the prism, the opposite face (here QR) being known as the *base.*

It is convenient to notice that the deviation is *towards the base,* or *away from the refracting edge* of the prism.

The deviation is measured by the angle AOK.

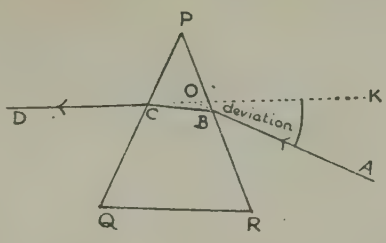

Fig. 96.
Refraction through Prism.

It is so important to be able to trace, graphically, the course of a ray of light through a prism, that an example worked out in full will now be given.

Example 37. Construct the path of a ray of light through a right-angled isosceles prism (refractive index 1.5) if the original direction of the ray is parallel to the hypotenuse face. Explain briefly the successive steps in the construction.—*Lond.* 1922.

Let PQ be the ray incident at Q (parallel to AB).

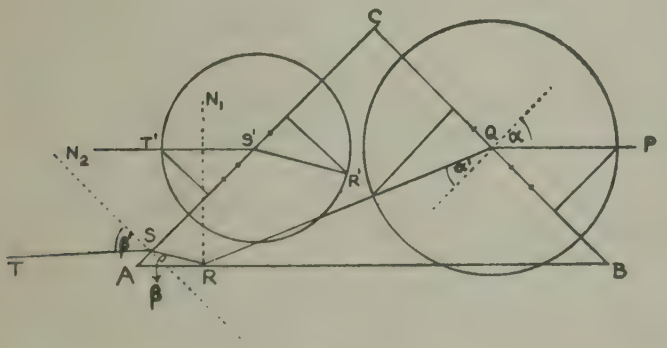

Fig. 97.

Following out the construction of §133, we obtain QR, the direction of the refracted ray. By measurement we find that QR is incident on AB at an angle of 73°, i.e., greater than the critical angle. Hence QR is reflected along RS. The triangle ARS is too small for accurate graphical construction, but we can draw a line, R′S′, parallel to RS and nearer to C. Repeating the "circle" construction (but keeping in mind that we are now working from glass to air) we obtain S′T′, and the emergent ray ST is, of course, parallel to this. If we may use a table of sines, the work is much simplified. Thus $\alpha = 45°$

$\frac{\text{Sin } \alpha}{\text{Sin } \alpha'} = 1.5.$

$\therefore \frac{\sin 45°}{\sin \alpha'} = 1.5.$

This gives sin $\alpha' = .471.$

$\therefore \alpha' = 28°.$

Hence RQB $= 90° + 28° = 118°.$

But QBR $= 45°.$

$\therefore$ QRB $= 180 - (118 + 45) = 17°.$

Hence $QRN_1 = 90 - 17 = 73°$, and as this is greater than the critical angle, QR is reflected. An easy calculation shows that the reflected ray RS is incident on AC at an angle β of 28°.

The angle β' is then worked out from the relation $\frac{\sin \beta'}{\sin \beta} = \frac{1.5}{1}$

Sin β = sin 28° = .47.

Substituting, we get sin $\beta' = .71$, whence $\beta =$ **45°**.

142. Minimum Deviation. Suppose P_1 is a pin two or three inches from a prism. An eye placed at E sees P_1 in the direction of P_1', owing to the refraction of the

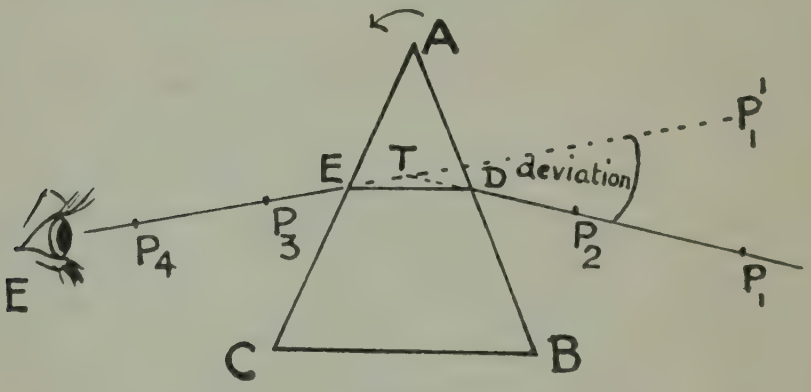

Fig. 98.

rays of light by the prism. Now let the latter be slowly rotated, say in the direction indicated by the arrow. P_1' moves nearer to P_1, i.e., the deviation (measured by the angle P'_1TP_1) becomes less. As we continue to rotate the prism, P_1' moves more and more slowly, and then begins to turn back. Just before this happens, i.e., when P_1' is as near as possible to P_1, the prism is said to be *in the position of minimum deviation.*

When in this position, place other pins, P_2, P_3, P_4, so that all appear to be in the same straight line, i.e., the line EP_1'. Join P_1P_2 and P_4P_3, meeting at T. Then P_1TP_1' is the angle of minimum deviation.

To obtain more accurate results, we should arrange that throughout the experiment the prism receives a movement of rotation only. To secure this we may mount it on a small cardboard circle—the circumscribing circle—so that A, B and C are on its circumference. An equal circle is drawn on a sheet of paper fixed on the working bench, and then in all movements of the prism we see to it that the two circles always coincide.

143. Formula involving Minimum Deviation. If D = angle of minimum deviation, A = angle of prism ($\angle$BAC in fig. 98), it can be proved that $\mu = \dfrac{\sin \frac{1}{2}(D+A)}{\sin \frac{1}{2} A}$ where μ is the index of refraction of the material of the prism.

144. μ by Minimum Deviation. If we can measure D and A, the formula evidently enables us to find μ. D is obtained by the method of §142, and A by tracing the outline of the prism on paper and using a protractor (or A might be found by the "rotated mirror" method of §110).

145. Construction for Minimum Deviation. It can be shown that when a prism is in the position of minimum deviation, a ray of light passes through it symmetrically, so that in fig 99, AD = AE. Hence we can trace the path through a prism of a ray of light for which the deviation is to be a minimum. We draw the horizontal section, ABC, of the prism and cut off AD = AE. Being given the index of refraction, we construct the refracted ray, EG, by the method of §133. The

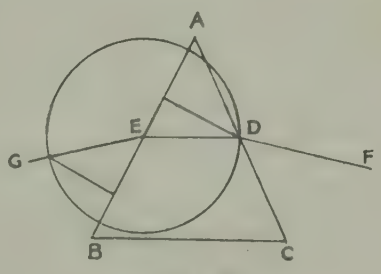

Fig. 99.
Construction for Minimum Deviation

incident ray is then easily drawn by making FDC = BEG. The complete construction is given in the figure, taking $\mu = 3/2$.

QUESTIONS ON CHAPTER VIII.

1. Describe an experiment to verify the laws of refraction, using a rectangular slab of glass and some pins.

Explain how it is sometimes difficult to see a crack in a glass tumbler when the eye is directly in front of it, but quite easy in other positions.—*Lond.* 1921.

2. A glass block is placed upon a sheet of printed paper, and the letters appear raised to an observer looking down through the glass. Account for this and describe how you would locate the apparent position of the letters. If the thickness of the block is 8 cms. and the refractive index of the glass 1.5, through what distance would the letters be raised ?—*Lond.* 1922.

3. Clear water 4 ft. deep seems 3 ft. deep to a person looking straight down into it, but less than this if he is looking obliquely. Explain these facts and quote the optical laws you have used in your answer.—*Lond.* 1926.

4. State the laws of refraction of light.

Describe in detail a method of determining the refractive index of water, proving any formula you may require for evaluating the result.—*O.C.* 1922.

5. When light has to be reflected through 90°, a right-angled prism is frequently used instead of a silvered mirror. Explain with diagrams how this is done, and why this method is preferred.—*Lond.* 1924.

6. When a clean empty glass test-tube is pushed down into a bowl of water, the submerged part often appears as though coated with silver. Why is this ?

Describe any experiment performed in the laboratory for finding the conditions under which the phenomenon occurs.—*M.L.L.S.B.* 1923.

7. State the laws of refraction of light, and describe experiments to verify them.

Explain carefully what you understand by the terms : Critical angle and total internal reflection.

Mention a practical application of total internal reflexion.—*Camb.* 1923.

8. Explain the statement that the refractive index from air to glass is 1.5.

Give a careful diagram of the path of a ray of light in passing from air through a rectangular block of glass 3 ins. thick, when the angle of incidence in air is 60°.—*Lond.* 1925.

9. Describe an experiment to determine the index of refraction of glass.

Show that when things are viewed obliquely through a window they do not appear in exactly their true positions.—*M.L.L.S.B.* 1926.

10. Explain what is meant by the refractive index and critical angle for light incident on a transparent material.

A ray of light falls normally on one face of a prism whose refracting angle is 30°. If the refractive index of the material be 1.5, draw carefully a diagram showing the path of the ray through the prism and find the deviation of the ray.—*Lond.* 1921.

11. Explain the terms: Critical angle and total reflexion.

The sides of a glass prism are each $2\frac{1}{2}$ ins. long, and a ray of light is incident normally on one face of the prism at a point 1 in. from the refracting angle. Draw a diagram showing the path of this ray through the prism and explain briefly your construction. (Refractive index of glass, 3/2.)—*Lond.* 1923.

12. A triangular glass prism (refractive index, 1.5) has three polished faces, the angles between the faces being 90°, 45° and 45°. A beam of light is directed normally on one of the faces enclosing the right-angle. Trace the path of the ray through the prism, giving reasons for your construction. For what purposes are such prisms employed?—*O.C.* 1921.

13. (a) Trace accurately the path of a ray of yellow light through a triangular glass prism when the angle of incidence is 45°. The angles of the prism are all 60°, and the index of refraction of the glass is $\frac{3}{2}$. Describe your construction in words.

(b) Measure the angles of, (i.) emergence, and (ii.) deviation. —*Oxf.* 1923.

14. What is meant by "refraction" and "refractive index"?

A ray of light travelling parallel to the base of an equilateral glass prism strikes one side. Trace the path of the ray through the prism by construction and find the deviation. Refractive index = 1·5—*C.W.B.* 1926.

15. How would you determine the deviation produced by a prism for light of a given colour, and what further observations

would you require to determine the refractive index of the prism ?

A ray of light coming through a hole in a shutter falls, after passing through a prism, on the opposite wall of the room. How will the position of the spot of light change as the prism is gradually rotated ?—*Camb.* 1921.

16. Show how you would use a prism, of which the section is a right-angled isosceles triangle, in order to bend a ray of light, (a) by refraction through the smallest possible angle, (b) at right angles.

In which of the above cases, and for what reasons, would you expect dispersion into colour to occur ?—*M.L.L.S.B.* 1925.

17. Construct a diagram showing the path of a ray of light through a glass prism of 60° angle, the deviation of the ray being a minimum. Mark the angle of deviation and state its value. The refractive index of the glass may be taken as 1.5.—*O.C.* 1924.

18. A ray of light is incident at an angle of 60° on a prism of 60°, of refractive index 1.5. Draw a diagram to scale showing the path of the ray through the prism, and measure the deviation produced. Find graphically the minimum deviation produced by the prism. (All constructional details must be shown and fully explained.)—*Camb.* 1924.

Chapter IX.

REFRACTION AT CURVED SURFACES

146. The Lens. A *lens* may be defined as **a portion of a refracting medium bounded by surfaces of a definite geometrical shape.**

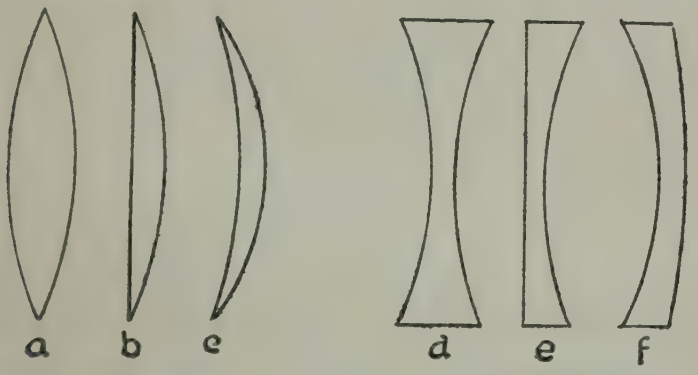

Fig. 100. Convex Lenses. Concave Lenses.

In the cases we shall consider, this shape will usually be a portion of a sphere (though one side may be plane). If the lens is thicker at the centre than at the circumference, it is convex ; if thinner, it is concave. Thus a, b, c represent convex lenses, while d, e, f represent concave ones.

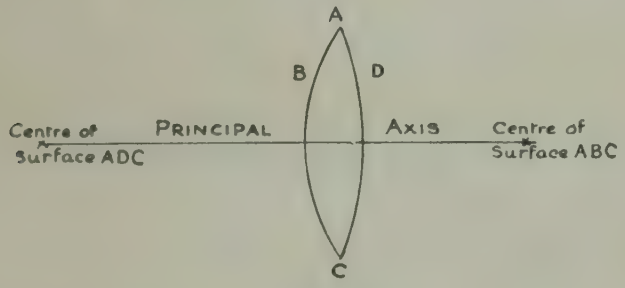

Fig. 101.

The straight line joining the centres of the two bounding spherical surfaces is known as the *principal axis* of the lens.

147. Optical Centre. Suppose a ray of light strikes a lens at A, traverses it by the path ACB, and leaves at B.

If the portions of the lens in the immediate neighbourhood of A and B respectively happen to be parallel, then that portion of the lens traversed by the ray may be regarded as a plate with parallel sides, and the emergent ray will be parallel to the incident ray, though not in the same straight line with it, i.e., there is some *displacement* (§140).

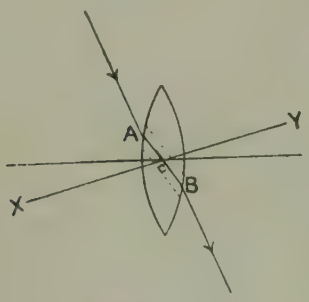

Fig. 102.
Optical Centre of Lens.

It can be shown that all rays of light which emerge from a lens in a direction parallel to their incident direction (such as ACB in fig. 102) pass through a fixed point on the principal axis. This point is known as the *optical centre* of the lens.

Conversely, all rays of light passing through the optical centre will emerge in a direction parallel to their incident direction. If the lens is thin (as in all the cases that we shall consider) the displacement may be neglected. Fu ther, in a thin lens we may regard the optical centre as being at any point within the lens on the principal axis.

In practice then, we shall assume that a ray of light such as XCY passing through C (any point within the lens, on the principal axis), travels as though the lens did not exist at all.

148. Action of a Lens. If we use the method of §141 to find the deviation produced by prisms of various angles, we soon discover that *the greater the angle of the prism, the greater is the deviation it produces.* This principle helps us to understand the action of a lens.

Suppose that in fig. 103a, ABCDE represents a convex lens divided into sections as shown. The faces of these sections are of course slightly curved, but if we take a large number of small sections, each one will be very nearly a portion of a prism, the angle of which decreases as we move from the outside portion, E, to the central

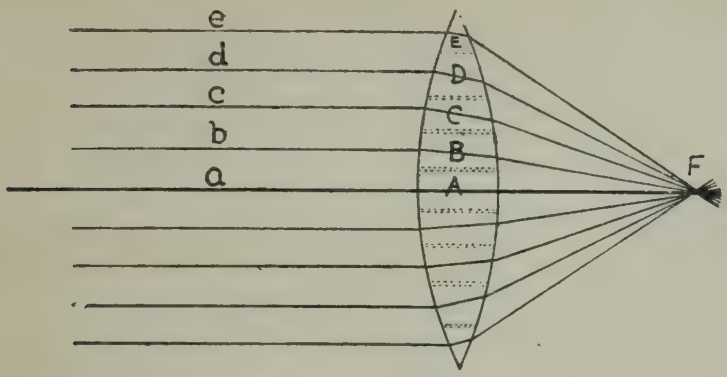

Fig. 103a. Action of a Convex Lens.

one, A (A is actually a portion of a parallel plate, which we may regard as a prism of angle 0°).

Now if we consider a number of parallel rays, a, b, c . . . . falling on such a prism, it is evident that the further a ray is from the centre, the more will it be deviated, because it is falling on a "prism" of larger angle. Hence it is quite possible for all the rays, after refraction, to meet in a single point, F, known as the *first principal focus* of the lens. The distance from the optical centre to the first principal focus is known as the *focal length* of the lens.

(N.B. For "second principal focus," see note at end of §151, and fig. 107.)

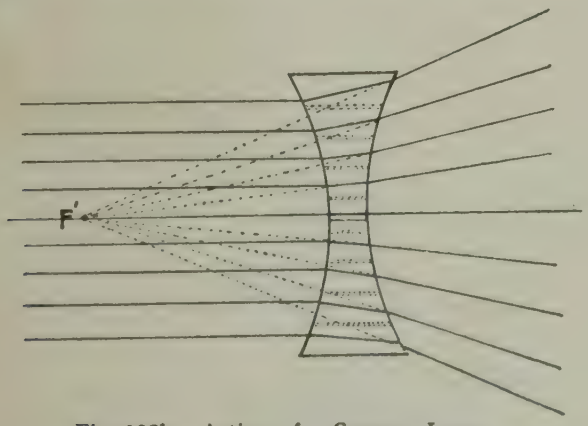

Fig. 103b. Action of a Concave Lens.

Remembering that a ray of light passing through a prism is always refracted towards the base, we

can see that a concave lens will act as illustrated in fig. 103b.

Here the rays of light after refraction will appear to diverge from F′, which is therefore a *virtual focus.*

149. Two Important Rays. We see from the last paragraph that :—

(i.) *A ray of light incident in a direction parallel to the principal axis will be refracted through a point known as the first principal focus.* Further, from §147, we have that

(ii.) *A ray of light passing through the optical centre proceeds without deviation.*

From (i.) and (ii.) we can easily trace out, by graphical methods, the size and position of the image formed by a given lens.

A simple case for a convex lens is worked out in fig. 104 below.

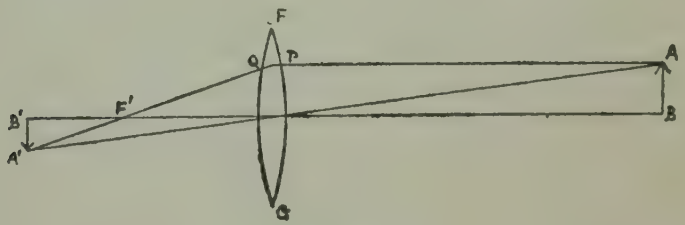

Fig. 104.

N.B. Refraction, of course, takes place at *each* surface (e.g. at P and Q in the figure). This gives an awkward construction, and it is therefore usual to draw a middle line, FG, and assume that refraction takes place only on the line. With a thin lens the error introduced is not appreciable.

150. Two Important Formulae. Let C be the optical centre of a concave lens, CA its principal axis, F its first principal focus (virtual), and B a point near the axis.

The ray, BD, parallel to the principal axis, is refracted along DE where FDE is a straight line.

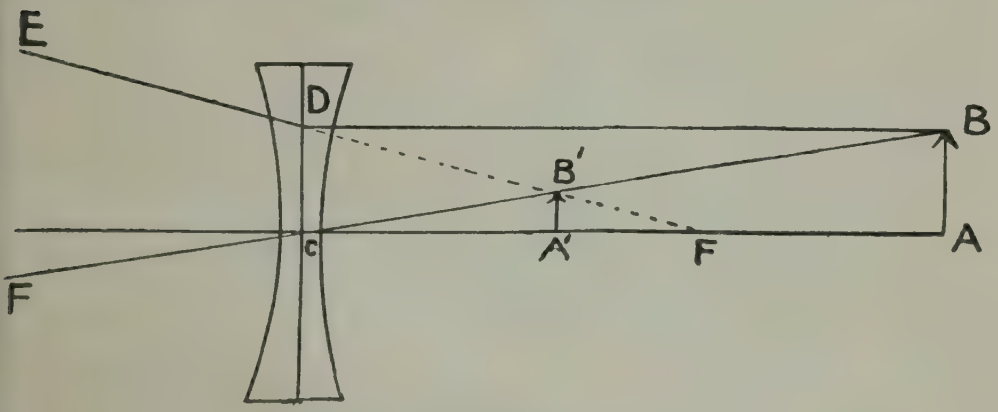

Fig. 105.

The ray BC undergoes no deviation. These two rays after refraction intersect at B′, which is therefore the image of B. Draw B′A′ perpendicular to the axis, cutting it in A′.

Then A′B′ is the image of the object AB.

Let CA = u, CA′ = v, CF = f (cp. §121).

By similar triangles,

$$\frac{u}{v} = \frac{CA}{CA'} = \frac{AB}{A'B'} = \frac{CD}{A'B'} = \frac{CF}{A'F} = \frac{f}{f-v}$$

Multiplying across, $vf = uf - uv$.

Dividing throughout by uvf, we have

$$\frac{1}{u} = \frac{1}{v} - \frac{1}{f} \text{ or } \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Again, the magnifying power of the lens is A′B′/AB,

And A′B′/AB = CA′/CA = v/u.

Hence

$$\textbf{magnifying power} = v/u = \frac{\textbf{distance of image from lens}}{\textbf{distance of object from lens}}$$

Though these formulae have only been proved above for a concave lens, they are equally true for a convex

one if proper regard is paid to sign, i.e., all measurements are to be made from the lens, and counted as positive if opposite to the direction of the incident light, negative if in the same direction.

151. Focal Length of Convex Lens.

First Method. The image of a distant illuminated object (e.g., window-frames or the sun) is obtained on a screen by means of the lens. The distance from lens to screen then gives f, the focal length. (This follows from the fact that the incident light is parallel; cp. §126 (iii.a).)

N.B. Since the image is formed on the side of the lens opposite to the object, it follows that a measurement from lens to image is made in the *same direction* as the incident light, and therefore the focal length of a convex lens is *negative.*

Second Method. A straight chalk-line is drawn on the bench, and over this are placed a lighted candle, the lens and a cardboard screen. The candle is moved forward or backward along the line until a sharp image of the flame is obtained on the screen.

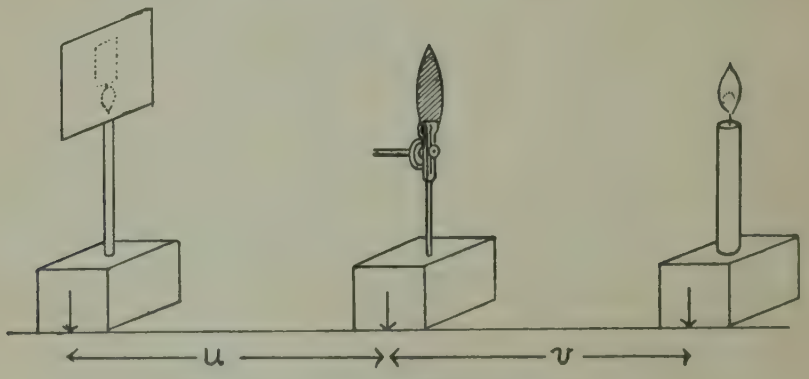

Fig. 106. Focal Length of Convex Lens.

Suppose distance from lens to candle = 24 ins., and distance from lens to screen = 8 ins.

Then $u = 24$ and $v = -8$.
Applying the formula, $1/v - 1/u = 1/f$,
We have $-\frac{1}{8} - \frac{1}{24} = \frac{1}{f}$,
Which gives $f = -6$.

A more accurate result is obtained if a piece of illuminated wire gauze is used instead of the candle (cp. §126 (ii.)).

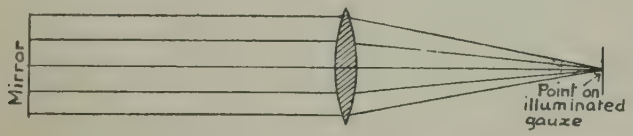

Fig. 107.

Third Method. The same arrangement is adopted as in the second method, except that a plane mirror is substituted for the screen, and a small piece of illuminated wire gauze (mounted on white cardboard) for the candle. The lens is moved until a sharp image of the gauze is formed just by the side of the gauze itself. The distance from the lens to the gauze then gives the required focal length.

For if the gauze is at a distance from the lens equal to the focal length, the rays of light will become parallel after refraction through the lens. These parallel rays strike the mirror normally and are reflected back along their own path, giving rise to an image practically coinciding with the original object. (N.B. In this case the object, or at least that point of it which lies on the principal axis, is said to be at the *second principal focus.*)

152. Example 38. The focal length of a convex lens is 12 ins. Draw a graph connecting u and v.

The lens being convex, the focal length is really *minus* 12 ins.
In the lens formula, $1/v - 1/u = 1/f$, we give a series of arbitrary values to u, and work out the corresponding values of v. Thus

if we decide to put $u=24$ we get $\frac{1}{v} - \frac{1}{24} = \frac{1}{-12}$, whence $v=-24$. We set out our results in a table, and then express them in graphical form (fig. 108).

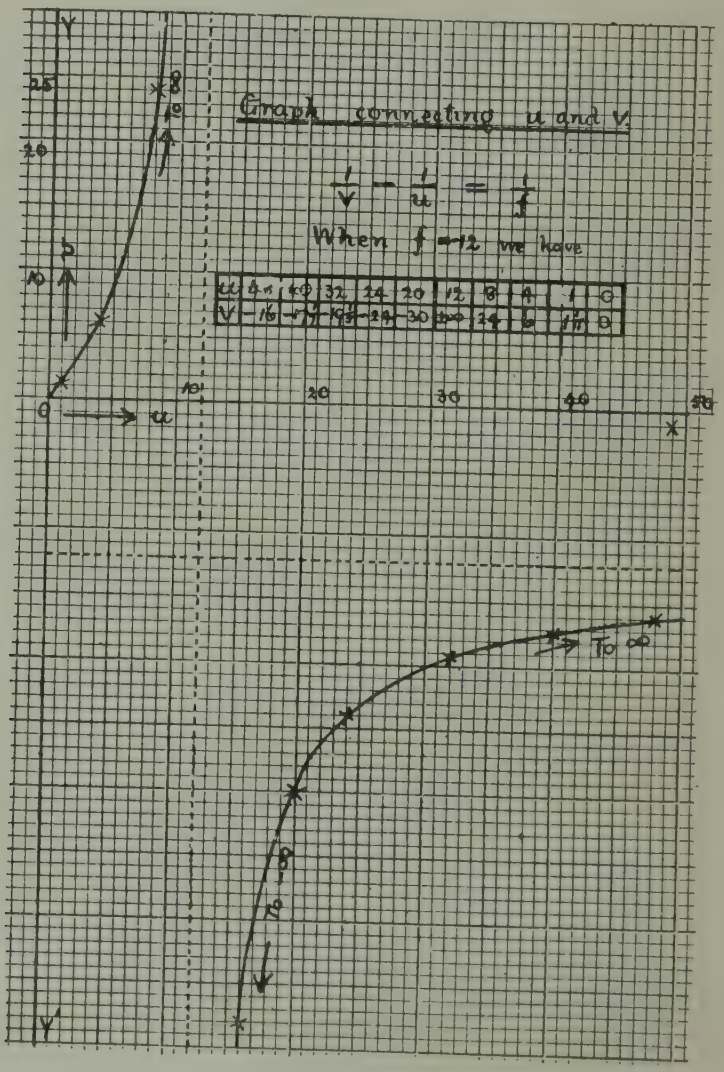

Fig. 108. Relation between u and v (Convex Lens).

Example 39. For the convex lens of the last example, i.e., focal length, 12 ins., find by careful drawing the nature, position and size of the image when a small object is at, (a) a distance

greater than the focal length, say 30 ins., (b) a distance equal to the focal length, (c) a distance less than the focal length, say 8 ins.

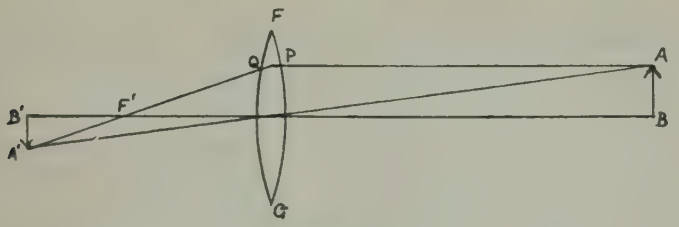

Fig. 109.

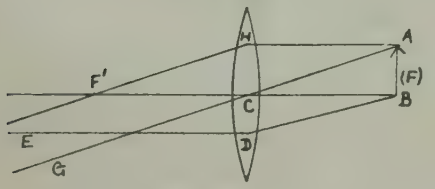

Fig. 110.

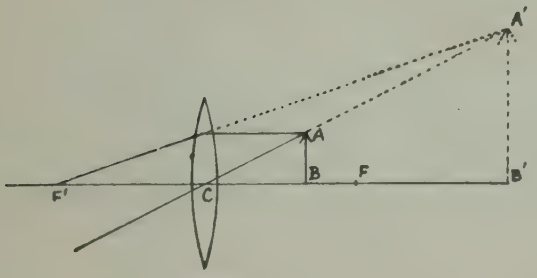

Fig. 111.

Case (*a*) is worked out in fig. 109. The image is real and inverted, .64 times the length of the object, and 19.2 ins. from the lens on the side opposite to the object.

(N.B. $\frac{\text{Length of image}}{\text{Length of object}} = .64$ and $\frac{\text{Distance of image}}{\text{Distance of object}} = \frac{19.2}{30} = .64$

The agreement between these two ratios is in harmony with §150).

Case (*b*). Here AB represents the object, B coinciding with F (fig. 110).

The ray, BD, is refracted along DE parallel to the principal axis (because this ray proceeds from the principal focus). The ray, BC, undergoes no deviation. Hence the refracted rays are parallel and the image of A is formed "at infinity," i.e., in practice no image is formed at all.

Similarly the refracted rays HF′ and CG (originally proceeding from A) are parallel.

Case (*c*) (fig. 111).

The image is virtual and erect, 3 times the length of the object and 24 ins. from the lens on the same side as the object.
(Here again we can verify the relation *magnification* $= v/u$.)

Example 40. Supposing that the image formed by a convex lens is the same size as the object when the latter is 10 ins. from the lens, find the position of the object in order that an erect image, magnified twice, may be formed.—*Lond.* 1920.

Since image is same size as object and $u = 10$, $\therefore v = -10$ (we can easily see that v could not be $+10$, for substituting these values of u and v in the formula, we should get $f = \infty$).
Substituting $u = 10$ and $v = -10$ in the formula, we obtain $f = -5$.

An *erect* image will be on the same side as the object (cp. fig. 111), and it is twice the size of the object, $\therefore v = 2u$. Hence from the formula we get, $\frac{1}{2u} - \frac{1}{u} = \frac{1}{-5}$ $\therefore u = 2\frac{1}{2}$. Thus the object must be placed **$2\frac{1}{2}$ ins. from the lens.**

Example 41. A convex lens has a small object placed on its axis distant from a focal point by an amount equal to one-quarter of the focal length. Prove that the image has linear dimensions 4 times as great as the object. Distinguish between the two possible cases.—*M.L.L.S.B.* 1921.

Let f = focal length.

The lens being convex, f is a negative quantity, $\therefore -f$ is a positive quantity. Also the distance of the object is a positive quantity (since in going from lens to object, we are moving opposite to the incident light). Its distance will be either $-\frac{3}{4}f$ or $-\frac{5}{4}f$, according as it is nearer to the lens than the focal point, or further from it.

In the first case we have $u = -\frac{3}{4}f$, and the formula gives

$$\frac{1}{v} - \frac{1}{-\frac{3}{4}f} = \frac{1}{f}$$

$\therefore v = -3f$.

Hence **magnification** $= v/u = \frac{-3f}{-\frac{3}{4}f} = 4$.

The magnification being $+^{ve}$, the image is **erect.**

In the second case, $u = -5f/4$, and substituting, we get

$$\frac{1}{v} - \frac{1}{\frac{-5f}{4}} = \frac{1}{f} \quad \text{whence } v = 5f.$$

$$\therefore \textbf{ magnification} = \frac{v}{u} = \frac{5f}{\frac{-5f}{4}} = \textbf{ -4.}$$

The magnification being —ve, indicates that the image is **inverted.**

Example 42. A pin, 4 cms. long, is set up 40 cms. from a convex lens of 15 cms. focal length. Where should a convex mirror of 6 cms. focal length be placed in order to form with the lens a real image of the pin coincident with the object? Show by a diagram the formation of the image, tracing at least one ray through the system from a point of the object not on the axis.—*Camb.* 1922.

We first find where the image would be produced if there were no mirror.

$u = 40$, $f = -15$, $v = ?$

Substituting these values of u and f in the formula $1/v - 1/u = 1/f$ we obtain $v = -24$, i.e., the image would be formed 24 cms. from the lens, on the side remote from the object.

Now in order to secure that a real image of the pin shall be formed coincident with the object, we must arrange that the rays of light which have passed through the lens (and are converging to form an image 24 cms. from it) shall be reflected along their own path. Hence these converging rays must be normal to the mirror, and so must be directed to its centre of curvature. To secure this, the *centre of curvature* must be 24 cms. from the lens. But the radius of curvature of the mirror = 12 cms. (twice its focal length).

$\therefore$ the *surface* of the mirror must be $24 - 12 = $ **12 cms. from the lens.**

The diagram is as shown below.

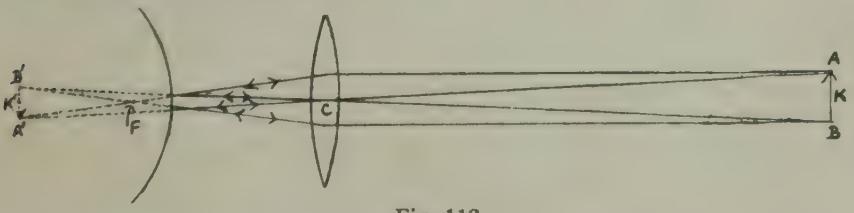

Fig. 112.

Note that since the image A′B′ is not a point, we cannot make the *whole* of it coincide with K′ the centre of curvature of the mirror. Hence the real image finally obtained will not coincide exactly with AB except as regards the middle point K.

QUESTIONS ON CHAPTER IX.

1. How is it that an image can be produced with a convex lens and not with two prisms placed base to base?

A convex lens has a focal length of 20 cms. Draw a graph on squared paper showing distances of a real object from the lens along the horizontal axis, and distances of the corresponding image along the vertical axis.—*Lond.* 1921.

2. What is meant by the focal length of a lens? Describe an experiment by which its value may be found for a given lens. —*Oxf.* 1924.

3. Define the principal focus of a lens.

It is required to project an image of an object magnified three-fold on to a screen 3 ft. away. What kind of lens must be employed, what must be its focal length, and where must it be placed?—*Lond.* 1923.

4. Name the chief properties of a convex lens. A candle flame is placed at the following distances, in turn, from a thin convex lens, whose focal length is 10 cms.: (a) 15 cms., (b) 10 cms., (c) 5 cms. State the result obtained in each case.—*Lond.* 1925.

5. A candle flame is 6 ft. from a screen. It is required to throw a real magnified image of the flame on the screen. Show with diagram how this may be done, (a) with a lens, (b) with a mirror. Find, graphically or otherwise, the position and focal lengths of the lens and of the mirror if the magnification in each case is to be 4.—*Lond.* 1926.

6. A convex lens has a focal length of 18 ins. An object 3 ft. high is placed at a distance of 6 ft. from the lens. What will be the size of the image?—*Camb.* 1923.

7. Distinguish between real and virtual images.

A convex lens has a focal length of 10 cms. Where must an object be placed in order that its image as formed by the lens may be three times the size of the object, (a) when the image is real, (b) when the image is virtual. Give diagrams.—*Camb.* 1924.

8. Explain what is meant by the focal length of a lens. On what factors does it depend?

Two convex lenses of focal length 6 ins. and 3 ins. respectively, are placed with their centres 10 ins. apart and their axes coincident. An object is placed 6 ins. in front of the lens of shorter focal length. Find the position of the final image, and make a careful diagram showing the path of a pencil of light through the system.—*Camb.* 1924.

9. Explain the terms *focus* and *focal length* as applied to mirrors and lenses.

It is required to produce a real image 3 times the size of an object, on a screen 100 cms. from the object. What kind of lens must be used, what must be its focal length, and where must it be placed?—*Camb.* 1925.

10. A beam of light from a distant source falls upon the surface of a solid cylindrical block of glass whose axis is at right angles to the direction of the beam. Trace the course of a single ray which does not pass through the axis. Under what circumstances would a pencil undergo no deviation whatever on passing through the glass? (Refractive index of air to glass, 1.5.)—*M.L.L.S.B.* 1922.

11. An object is placed 40 cms. from a convex lens, and a real image of the object is formed 40 cms. on the other side of the lens. A concave lens is placed 20 cms. from the convex lens on the side remote from the object, and the image is now found to be produced at a distance of 50 cms. from the convex lens. Draw a diagram to show how the images are formed, and calculate the focal lengths of the two lenses.—*O.C.* 1922.

12. State the laws of refraction and describe some method of demonstrating the truth of these laws.

Find graphically or by calculation the deviation of a ray of light which passes in a horizontal plane through a vertical glass cylinder, the angle of incidence being 45°. (Refractive index of glass = 1.5.)—*Camb.* 1925.

13. Explain what is meant by focal length of a convex lens.

An electric light is 6 ft. from a wall, and it is found that a convex lens held 2 ft. from the light throws a sharp image on the wall. In what other position may the lens be placed so as to throw a sharp image on the wall? Give reasons. Contrast the appearances of the images and calculate the focal length of the lens.—*C.W.B.* 1924.

Chapter X.

DISPERSION, COLOUR, OPTICAL INSTRUMENTS.

153. Production of Spectrum. About the year 1670, Sir Isaac Newton performed an experiment in which a pencil of sunlight entered a darkened room through S, a small hole in a shutter. A patch of light, AB, was received on a screen, but when a prism was placed in

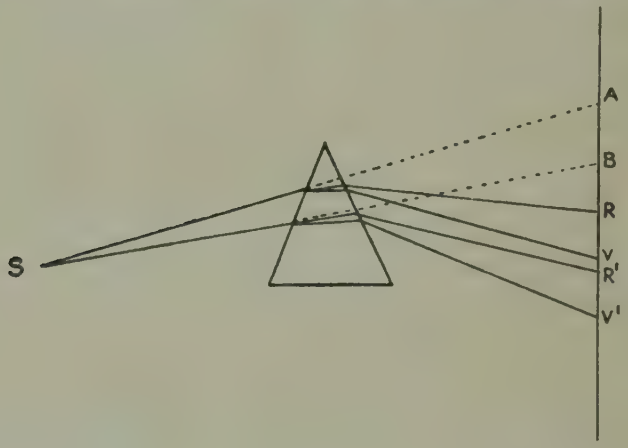

Fig. 113.

the path of the beam a coloured band, RV′, was obtained in the position shown in fig. 113.

Explanation. The "white" light that falls on the prism is really a mixture of light of many different colours. All the light is refracted by the prism, but not to the same extent—the red shows the least deviation and the violet most, other colours occupying an intermediate position. If the light incident on the prism were only red, we should have the red patch, RR′ on the screen, while if it were only violet we should have the violet patch VV′. But as the light contains both red and violet (and many intermediate colours), we get both RR′ and VV′ (and many intermediate patches). The net result is that the coloured band or spectrum is red at one end and violet at the other, with a confusion of colours in between, the confusion being caused by the

overlapping of many differently coloured patches. A spectrum such as this is said to be *impure*, because the colours are not clearly marked off one from another. This separation of light into different colours is known as *dispersion*

154. Virtual Spectrum. S is a vertical slit about 1 mm. wide in a sheet of cardboard or (better) copper. It is illuminated from behind by a gas flame, and it is

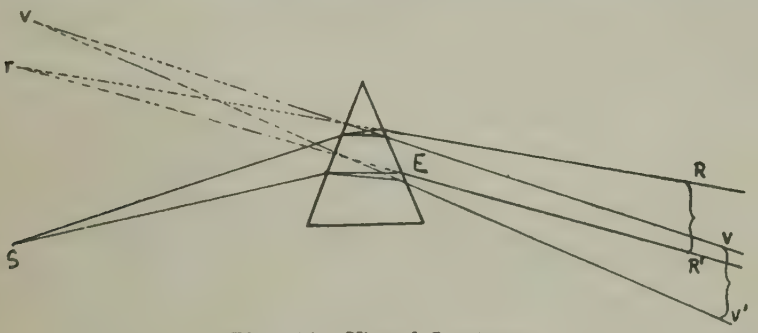

Fig. 114. Virtual Spectrum.

observed through a prism held with its edge parallel to the direction of the slit, i.e., vertical, the eye being placed at E. A spectrum, *rv*, is observed, and by gently turning the prism this spectrum can be brought to a minimum distance from S, i.e., into the *position of minimum deviation*. In this position it is found to be more clearly defined than in any other.

By examining fig. 114, we can see how this spectrum is formed. The red constituent of the incident light becomes the pencil RR', which appears to diverge from *r*. Thus at *r* we have a red image of the slit. Similarly at *v* we have a violet image, and in between we can make out orange, yellow, green and blue images. Since each colour has its own position, the spectrum is in this case pure.

The reason for the "minimum deviation" position should be noted. It can be proved that rays of light proceeding from a point will not (after refraction through a prism) appear to come from a point, unless the prism

is in the position mentioned. Hence it is only in this position that clearly defined (virtual) images of the slit will be produced.

Note also the importance of a *narrow slit.* A wide slit would give us wide images, which would to some extent overlap and impair the purity of the spectrum.

155. Real Pure Spectrum. In §153 we obtained a real but impure spectrum, and in §154 a pure but virtual one. We shall now see how to obtain a real pure one.

S is a vertical, narrow, illuminated slit (as in §154), and by means of a convex lens, L, an image, S′, of the

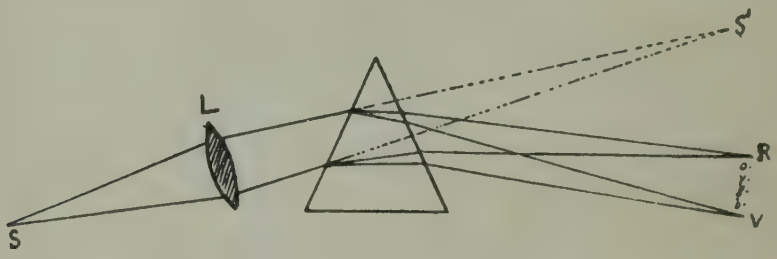

Fig. 115. Real Pure Spectrum.

slit is focussed on a screen. The prism is now placed between L and S′ with its edge parallel to the length of the slit. A spectrum, RV, is now obtained, and the prism is then gently rotated until RV is in the position of minimum deviation.

To avoid confusion, only the refracted rays giving rise to R and V have been shown in the figure, but the other coloured images lie in between. Note that there are not just *seven* spectral colours. Though the eye can only distinguish about seven, these shade off into one another, so that the number of colours is really infinite. Colour, in fact, corresponds to wave-length (§95), and there are as many shades of colour as there are differences of wave-length.

To obtain a real pure spectrum then, the necessary conditions are :—

1. A **convex lens** to enable us to obtain real images of the slit.

2. A **narrow slit,** so that the real images may not be so wide as to overlap one another.
3. **Prism in the position of minimum deviation,** because otherwise it is not possible to obtain sharply defined images of the slit.

(N.B. When the prism is in this position for the red, it is "out" for the violet and vice versa. It should be so adjusted as to be right for the middle portion of the spectrum.)

156. Recomposition of White Light. Since white light can be split up into the colours of the spectrum, it should be possible to recombine these colours into white light. We can do this :—

(1.) By placing a second prism—exactly similar to the first, but with its refracting edge in the opposite direction—so that it receives the light which has been decomposed by the first one, as in fig. 116.

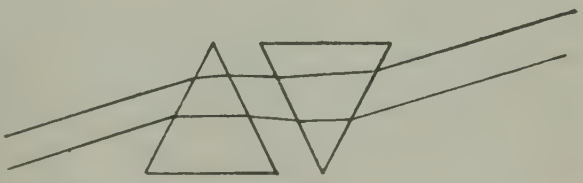

Fig. 116. Recomposition of White Light.

The two prisms are together equivalent to a single plate of glass with parallel sides.

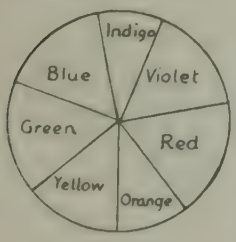

Fig. 117. Newton's Disc.

(2.) By Newton's Disc. This is a circular disc divided into coloured sectors corresponding to the colours of the spectrum. It is attached to a whirling table, and when rapidly rotated appears white, or at least grey. The reason is that a visual impression persists on the retina of the eye for about 1/10 second after the object causing that impression has been removed. Hence the whole succession of colour impressions is received before the first of them has died away. The coloured sectors, however, absorb a considerable amount of light, as will be explained in the next paragraph. Hence the light

received by the eye is deficient in illuminating power, and that is why the disc looks grey instead of white.

157. Colour due to Selective Absorption. If a narrow strip of *white* paper is placed successively in different parts of the spectrum, it appears red in the red part, blue in the blue part, and so on. A piece of *red* paper, however, only appears red in the red part of the spectrum, elsewhere it appears black. Similarly a piece of blue paper appears black in every part of the spectrum except the blue.

These and many other results are easily explained if we consider that *the colour of a body is the colour of the light which it reflects.* A white object can reflect light of *every* colour, and so the strip of white paper in the experiment reflects red light when placed in the red part of the spectrum, blue light when placed in the blue part, and so on. A red object, however, is only capable of reflecting red light. Hence the strip of red paper when placed in (say) the blue part of the spectrum appeared black, because it was incapable of reflecting any of the light (not being red light) which fell upon it. Instead of reflecting it, it absorbed it.

In one respect the above statements need a little qualification. The colour of a body is seldom "pure," for instance, a red body usually reflects a little yellow light as well as red. Hence the "red" strip of paper would not only appear red in the red part of the spectrum, but would appear yellow in the yellow part. Elsewhere it would appear black. Similarly a blue object often reflects some green light as well as blue, while a yellow object may reflect green in addition to yellow. There is a general tendency, in fact, to encroach on a neighbouring part of the spectrum.

158. Complementary Colours. Suppose we make a Newton's Disc (§156), and then cut out the yellow sector. On making the remaining part of the disc revolve, the effect of blue is produced. Thus white light may be

regarded as made up of yellow and blue, and these colours are therefore said to be *complementary*.

If we begin again with a complete disc, and cut out the red, the remaining portion, when revolved, gives us a greenish-blue effect. Hence we may say that red and greenish-blue are complementary. In a similar way it is possible to find out many other pairs of colours which are complementary.

159. Applications. The last two paragraphs provide the material for working out many problems connected with colour. We can understand, for instance, why a gas flame appears nearly invisible when looked at through two glasses of certain different colours, say red and blue. The first glass absorbs every colour except red, and therefore only red light falls on the blue glass. But the latter absorbs every colour except blue. Hence it will absorb the red light which falls upon it—and in this case the red light is *all* the light.

In the red light of a photographer's dark room, the green baize of the printing flame appears black. For the green baize absorbs every kind of light except green, which (ordinarily) it reflects. In this case it absorbs all the red light which falls on it.

Remembering what was said about complementary colours, it seems strange that when a painter mixes the complementary colours blue and yellow he should obtain, not white, but *green*. The explanation is that the blue pigment absorbs all colours except blue and green (cp. §157), while the yellow pigment absorbs all colours except yellow and green. Thus a mixture of the two pigments absorbs all colours except green, which is therefore the colour of the mixture.

Many problems in colour are connected with the subject of wave-length. For instance, why does the sun appear red when seen through smoke or fog? The tiny particles of soot, or water, are large enough to act as obstacles to the shorter waves of light (constituting the blue-violet end of the spectrum), but not to the longer waves (red

end of spectrum). Hence the short waves are turned back, while the longer waves succeed in reaching the observer.

Similarly the blue appearance presented by the smoke of a chimney is due to the shorter waves being reflected to the observer.

160. Inconvenience of Dispersion. Dispersion (i.e., the separation of light into its constituent colours), may be a source of great inconvenience. We have seen (§148) that a lens may be regarded as made up of a great number of prisms, or rather portions of prisms. Hence light, on passing through a lens, will be dispersed, and as the violet rays undergo the greatest amount of refraction, the "violet" focus will be nearer to the lens than the "red" focus. Fig. 118 illustrates this for a convex lens, the amount of dispersion being exaggerated for the sake of clearness. Between V and R are, of course, the various other foci, not shown.

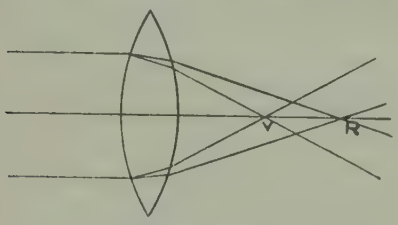

Fig. 118.
Dispersion produced by Lens.

If a cardboard screen is placed at V, it will be evident from the figure that the image will have a red border, while if it is placed at R the border will be violet. There is a point between V and R where the colour of the border changes from red to violet, and (if the incident light is parallel) this point is taken as the true position of the focus.

The fact remains, however, that the image has a coloured border, and a lens giving such an image would be quite unsuitable for the construction of a good telescope or microscope. The defect is known as **chromatic aberration,** and is overcome as follows.

Suppose that next to the convex lens which shows the aberration, we place a concave lens of the same material and of equal power. The concave lens (like the reversed prism of §156) causes the dispersed rays to recombine into white light. The colour effect is destroyed,

but unfortunately so is the image! The combination of lenses, like the pair of prisms of §156, acts like a plate with parallel sides.

The solution of the problem depends on the fact that if two lenses of equal focal length be made, a convex one of crown glass and a concave one of flint glass, the flint glass will produce the greater amount of dispersion. Hence if the curvature of the concave lens be reduced—making it "weaker" as a lens and at the same time reducing its power of causing dispersion—a point will be reached at which it will be able just to neutralise the dispersion produced by the convex lens, and evidently it will now be weaker than the convex lens. Hence the combination will be **achromatic** (i.e., it will produce no colour effect), but it will act as a convex lens, weaker of course than the single convex lens would be if uncombined with a concave one. The action of the compound lens is illustrated by fig. 119.

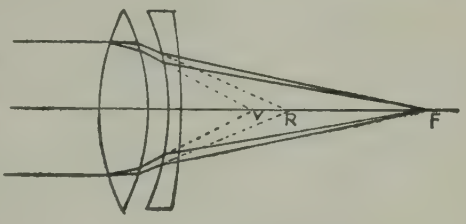

Fig. 119.
An Achromatic Combination.

OPTICAL INSTRUMENTS.

161. Optical Lantern. This consists essentially of a small but powerful source of light, K, the light from which passes through the condenser, C, usually consisting

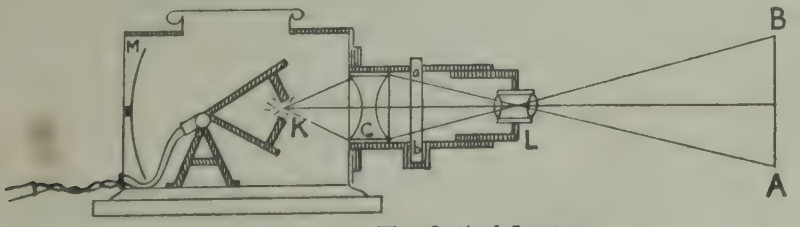

Fig. 120. The Optical Lantern.

of two plano-convex lenses (the combination acting as a convex lens). If K is at the second principal focus of C (§151, end), the light emerging from C is parallel. By a

screw movement K may be drawn back a little, when the rays of light that have passed through Ç will be convergent. They then fall on the slide, *ab*, in front of which is another convex lens, L (or rather a system of convex lenses) called the *objective*. This is mounted in a brass tube which can be moved forward or backward by means of a screw. A real inverted image, AB, of the slide may be sharply focussed on the screen.

The concave mirror, M, serves to reflect on to the condenser some light that otherwise would have been absorbed by the back of the box.

162. The Photographic Camera. Here we have to consider :—

(1.) The object to be photographed, AB.
(2.) The (convex) lens, L.
(3.) The ground-glass screen, S.

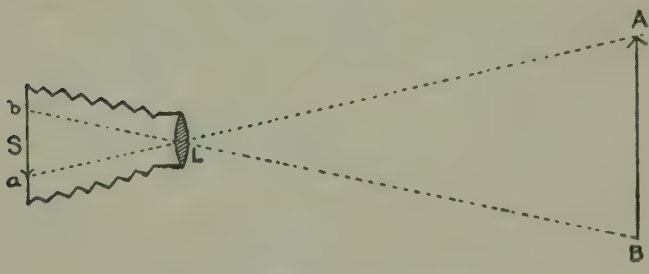

Fig. 121. Photographic Camera.

When AB is at a good distance (theoretically infinity), and S is at the first principal focus of L, a sharp image is formed on S. The lens can be moved forward or backward, usually by a "bellows" arrangement, until the image is in focus, and the screen is then replaced by a sensitive plate or film.

By comparing figs. 120 and 121, it will be seen that the camera is in some respects an inversion of the optical lantern.

163. The Eye. This is shown diagrammatically in fig. 122. In essential principles it is very similar indeed to the photographic camera. Note :—

(i.) The *sclerotic*, the tough, opaque outer coating, spherical in shape except at the front, where it projects, forming

(ii.) The *cornea*, which is transparent so as to admit the rays of light;

(iii.) The *crystalline lens*, a convex lens of which the curvature, and therefore the focal length, can be altered by means of the *ciliary muscle*;

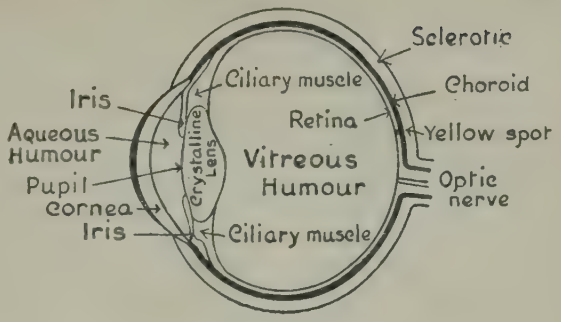

Fig. 122. The Eye.

(iv.) The *iris* (the coloured portion of the eye), really a ring-shaped curtain, of which the "strings" (muscular fibres) are arranged along the radii. Under the action of strong light, the fibres lengthen and the iris broadens, thus reducing the amount of light which passes through the circular aperture known as the *pupil*;

(v.) The *retina*, a fine network extension of the optic nerve (which passes from the back of the eye to the brain). Images are received on the retina, the most sensitive part of this being the *yellow spot*.

When the eye is at rest, distant objects are focussed on the retina, and so it seems as though a *near* object would give a blurred image, because the positions of the lens and the retina are fixed. (Contrast the camera, in which the lens can be moved.) Hence the need of the change of curvature already mentioned. This power of adjustment possessed by the eye is known as *accommodation*.

164. Nearest Distance of Distinct Vision. The accommodative power of the eye is not unlimited, however, and a sharp image can be obtained on the retina only when the object is at a certain minimum distance from the eye, known as the *nearest distance of distinct vision*. For persons of normal sight this distance is about 10 or 12 inches.

165. Common Defects of Vision. Two of the commonest defects are:—

(i.) *Short Sight* or *Myopia*. In this case the axis of the eye is a little too long from front to back. Hence

the rays of light come to a focus in front of the retina instead of on it. The trouble is corrected by putting a suitable concave lens in front of the eye. The action of this will be clear from an examination of figs. 123 and 124.

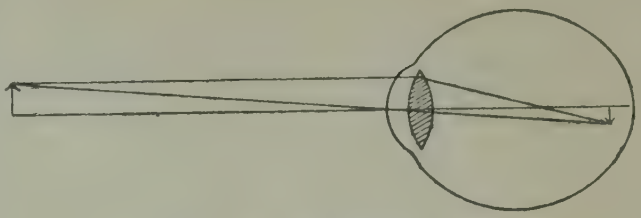

Fig. 123. Short-sighted Eye.
Image formed in front of Retina.

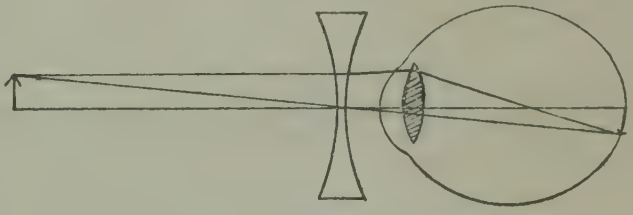

Fig. 124. Short-sighted Eye, corrected.

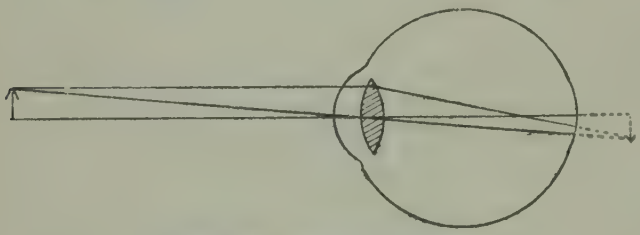

Fig. 125. Long-sighted Eye.

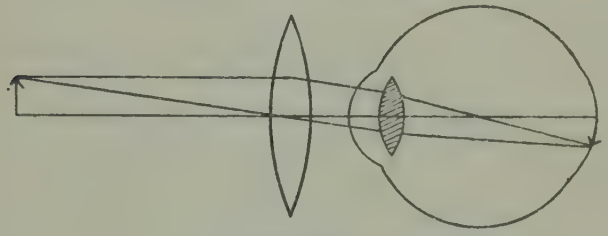

Fig. 126. Long-sighted Eye, corrected.

(ii.) *Long Sight* or *Hypermetropia.* This defect is often met with in middle-aged persons, who find themselves obliged to hold a book or newspaper at a good distance from the eye in order to read it. The trouble is that with advancing age the accommodative power of the eye has declined—the crystalline lens can no longer assume sufficient curvature for near objects. In this case a suitable convex lens must be supplied. See figs. 125 and 126.

166. Lens required for Short-sighted Eye. We have just seen that a concave lens is required. We must now consider the exact focal length required in a given case.

Suppose the greatest distance at which a short-sighted person can see an object distinctly is 8 inches (i.e., his "far-point," as it is called, is at this distance). To correct his vision, he needs a concave lens such that an object which is really "at infinity" will seem to him, after the rays of light have been refracted by the concave lens, to be at a distance of 8 inches. Hence in the usual lens formula, $1/v - 1/u = 1/f$, we have $u = \infty$ and $v = 8$. $\therefore f = 8$. Hence for a short-sighted eye, a person requires a concave lens with a focal length equal to the distance of his far-point.

167. Lens required for Long-sighted Eye. A long-sighted person can usually see distant objects distinctly, but the rays from nearer objects are brought to a focus behind the retina instead of on it. Suppose he wishes to be able to read a newspaper comfortably at a distance of 10 inches (the nearest distance of distinct vision for a normal person), and that *his* nearest distance of distinct vision is 40 inches. He requires a lens such that the rays of light from an object 10 inches away will, after refraction through the lens, appear to diverge from a point 40 inches away.

Hence in the formula, $1/v - 1/u = 1/f$,

we substitute $u = 10$ and $v = 40$,

obtaining $1/40 - 1/10 = 1/f$, which gives $f = -13\frac{1}{3}$.

Thus a convex lens of focal length $13\frac{1}{3}$ inches would be required.

168. Simple Microscope (or **Magnifying Glass**). This consists of a convex lens which is held in front of the object to be examined (small print, etc.) at a distance a little less than its own focal length. This gives an erect virtual image, as illustrated in fig. 127.

It is not obvious at first that with this arrangement we have any magnification. Certainly the image A′B′, is larger than the object AB, but according to the figure it does not *seem* any larger, because it is proportionally further from the eye. (This statement requires a little

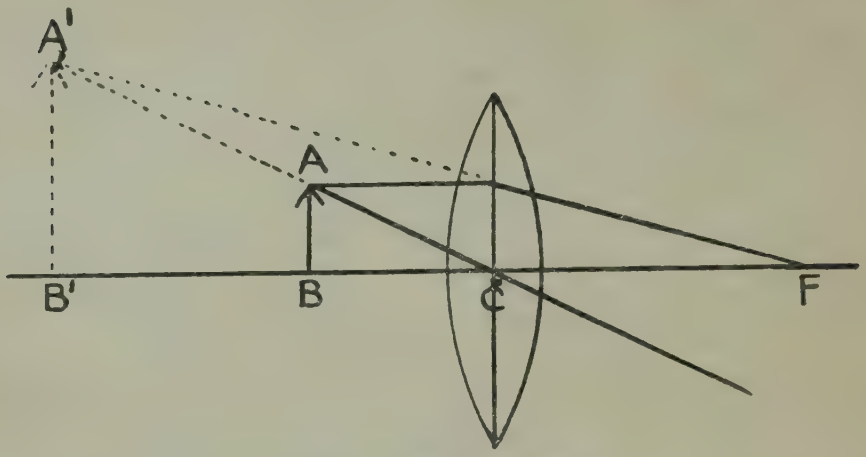

Fig. 127. Simple Microscope.

qualification. A'B' : AB = B'C : BC, so that it would be quite true only if the eye were actually at C.)

We must remember, however, that when we are using the lens to the best advantage, it is adjusted so that A'B' is *at the nearest distance of distinct vision.* Hence if we had no lens, the best we could do with the object AB would be to place it in the position occupied by A'B', because we could not see it distinctly if we placed it any nearer. Therefore by means of the lens we obtain an image of length, A'B', in the same position as (if we had no lens) would be occupied by the object AB.

$$\text{Hence magnification} = \frac{\text{length of image}}{\text{length of object}} = \frac{A'B'}{AB} = \frac{B'C}{BC}$$

$$= \frac{\text{distance of image.}}{\text{distance of object.}}$$

A convenient approximate formula is obtained by noting that BC is approximately equal to the focal length of the lens. Our formula then becomes :—

$$\text{Magnification} = \frac{\text{nearest distance of distinct vision}}{\text{focal length of lens}}$$

In working out a problem on the magnification of a simple microscope, we are guided by the terms of the question as to which formula we should use.

Example 43. How must a lens of 5 cms. focal length be placed in front of a small object so as to form an image 25 cms. from the lens on the same side? What is the magnification in this instance?—*Lond.* 1924.

Here $f = -5$, $v = 25$, $u = ?$
Substituting in the formula, $1/v - 1/u = 1/f$,
We obtain $u = 4\frac{1}{6}$.
Hence **magnification** $= v/u = 6$.

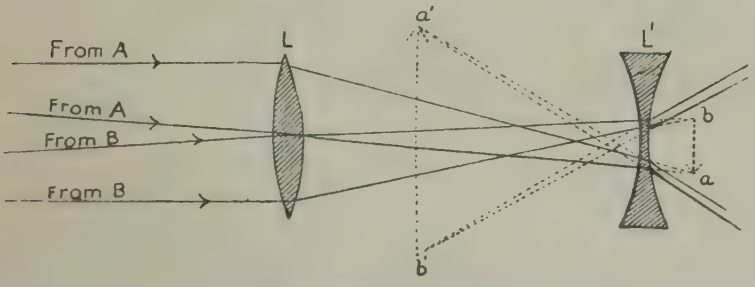

Fig. 128. Galileo's Telescope.

169. Galileo's Telescope. The action of this instrument will be understood by reference to fig. 128. The "object glass" consists of a convex lens, L, and this would produce a real inverted image, *ab*. The rays are, however, intercepted by a concave lens, L′, so that an erect virtual image, *a′b′*, is actually produced. The concave lens fits into a sliding tube, and it can be moved so that *a′b′* is at the nearest distance of distinct vision from the eye.

This telescope (duplicated) forms the ordinary opera glass. It possesses the advantage of giving an erect image and of being short, for the distance between the lenses is obviously less than the focal length of the object glass.

(Contrast astronomical telescope in both these respects.)

170. Astronomical Telescope. The object glass is a convex lens as before, and in this case produces a real, inverted image, *ab*, in front of the "eye-piece"—a convex lens of much shorter focal length than the object glass.

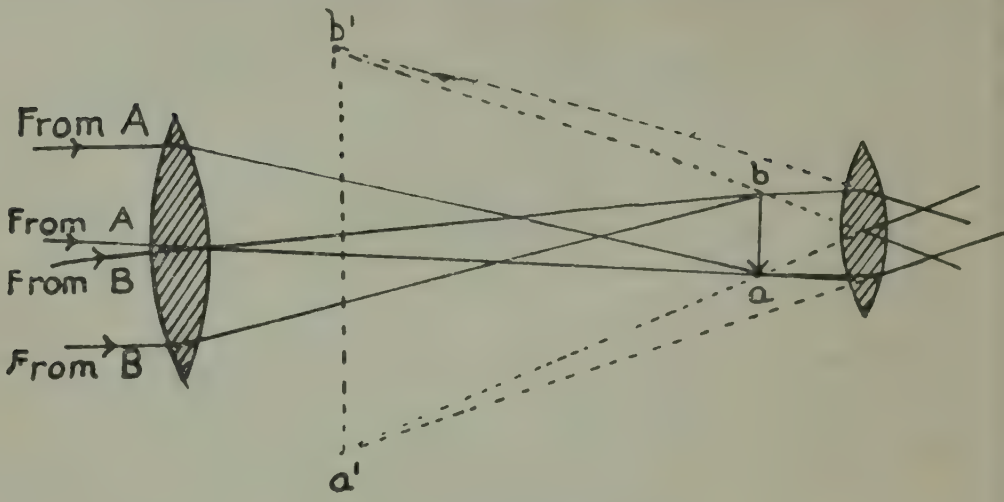

Fig. 129. Astronomical Telescope.

The distance of the eye-piece from the real image is a little less than its own focal length, and a virtual image is formed at the nearest distance of distinct vision. The image is inverted with respect to the original object, but in astronomical work this does not matter.

Note that the relation of the eye-piece to the real image is the same as that of the simple microscope to the small object which it is being used to examine.

In the case both of the astronomical telescope and of Galileo's telescope, it can be shown that :—

$$\text{Magnification} = \frac{\text{focal length of object glass}}{\text{focal length of eye-piece}}$$

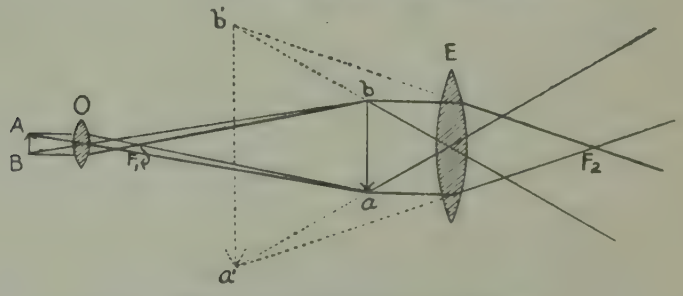

Fig. 130. Compound Microscope.

171. Compound Microscope. It will be seen from fig. 130 that the principle of the compound microscope is

very similar to that of the astronomical telescope. The chief differences are that :—

(i.) The object glass (or "objective" as it is called in the case of the microscope) is of very short focal length, and

(ii.) Special arrangements have to be made to secure brilliant illumination of the object ; otherwise (owing to the high magnification) the image would be too faint to be visible.

As in the case of the simple microscope, the magnification is equal to the ratio

(length of image) : (length of object).

Example 44. Two convex lenses of focal length 4 cms. and 15 cms. are combined to form a microscope. An object is placed 6 cms. from the first lens, and the final image is formed 25 cms. from the second lens. Show its formation by a diagram, tracing at least two rays from a point on the object to the eye, and calculate the magnification.—*Camb.* 1923.

The diagram is a special case of fig. 130, in which, however, a′b′ will come slightly to the *left* of O.

Distance from AB to O = 6 cms., and from optical centre of O to F_1 is another 4 cms. We can therefore draw the rays which define the image ab.

The diagram would show that this image is 12 cms. from O. Or, by calculation, $u = 6$, $f = -4$, and therefore from $1/v - 1/u = 1/f$ we have $v = -12$.

We now calculate the position of the second lens, E. We know that $v = 25$ and $f = -15$, whence, by the usual formula, we have $u = 9\frac{3}{8}$. Thus the second lens is $9\frac{3}{8}$ cms. from ab, or $12 + 9\frac{3}{8} = 21\frac{3}{8}$ cms. from O. We can now insert the second lens E in correct position in our diagram and complete the drawing, obtaining a′b′, which will be found to be 25 cms. from E.

To obtain the magnification, we have ab/AB = 12/6 (§150).

∴ ab = 2 AB. Again a′b′/ab = 25/$9\frac{3}{8}$ (§150). ∴ a′b′ = $2\frac{2}{3} \times$ ab = $2\frac{2}{3} \times 2$ AB.

∴ a′b′ = $5\frac{1}{3}$ AB.

∴ **magnification** = a′b′/AB = $5\frac{1}{3}$.

QUESTIONS ON CHAPTER X.

1. If a beam of white light is passed through a prism and then intercepted by a screen, we see on the screen a patch of white light with coloured edges. Explain this with the help of a diagram, and show how to modify the experiment so as to obtain a clear spectrum on the screen.—*Lond.* 1922.

2. A narrow slit illuminated by white light is observed, (a) through a prism, (b) obliquely through a slab of glass with plane parallel faces. Explain with the help of diagrams any differences you would observe.—*Camb.* 1924.

3. What experimental arrangement would you adopt in order to produce a pure spectrum on a screen? Explain the function of each part of the apparatus.—*C.W.B.* 1925.

4. How may it be shown that white light can be split up into several coloured parts, which can be re-combined to form white light again?—*M.L.L.S.B.* 1923.

5. A slit illuminated by sunlight is viewed through a triangular glass prism. Describe and explain what you could see. How would the appearance be modified if a sheet of red glass is placed, (a) between the slit and the prism, (b) between the prism and the eye?—*Camb.* 1922.

6. On what does the apparent colour of objects depend? What colour would you expect this (green) examination paper to appear, (a) in a photographer's red light, and (b) in the light given by a sodium flame?—*Oxf.* 1924.

7. Write a short essay on "Colour," indicating briefly the experimental basis of any statements you make.—*Oxf.* 1923.

8. Explain, with the aid of a diagram, how a prism produces (a) deviation, (b) dispersion, in a beam of light.

Is it possible for a beam of white light to pass through a prism without being dispersed?—*Camb.* 1925.

9. Show that, owing to chromatic dispersion, it is impossible to get an absolutely clear image with a single lens; and explain how the difficulty can be overcome by using a composite one.—*M.L.L.S.B.* 1926.

10. What are the two focal points of a convex lens? What do you know about them?

A lens placed 6 ins. from an object produces a real image twice the size of the object. Where must the lens be placed to form an erect image also twice the size of the object?—*Lond.* 1927.

11. If you wished to throw on a wall a magnified image of a brightly illuminated object lying flat on a table, how could you do so with the aid of a lens and a plane mirror?

Show further the exact positions you would adopt to get the image magnified four times in diameter, using a lens of 2 ft. focus.—*M.L.L.S.B.* 1925.

12. Describe an optical lantern (or magic lantern) for producing pictures on a screen.

Why does the slide have to be put in upside down?

On what does the magnification depend?—*Oxf.* 1924.

13. Draw a diagram showing the structure of the human eye. Mark clearly on your diagram the following parts and explain briefly the function of each: Cornea, ciliary muscle, iris diaphragm, blind spot, yellow spot.—*Oxf.* 1924.

14. Explain the term *accommodation* as applied to the eye. What kind of spectacles are required by a person whose minimum distance of distinct vision is 36 ins., to enable him to read a book at a distance of 10 ins. Give diagrams to illustrate your answer. —*Camb.* 1921.

15. What is *accommodation* as applied to the eye, and how is it effected?

What spectacles are required by a person whose minimum distance of distinct vision is 6 ft., to enable him to read a book at a distance of 18 ins.?—*Camb.* 1925.

16. Describe the defects of vision known as "long-sight" and "short-sight," and explain how they may be corrected by lenses.

A man's shortest distance of distinct vision is 3 metres. What spectacle lenses does he require to enable him to read a book at a distance of 25 cms.?—*O.C.* 1925.

17. Give an account of two common defects of vision.

A person whose nearest distance of distinct vision is 20 cms. uses a lens of 5 cms. focal length to magnify a small object. What is the distance of the object when in focus, and what is the magnification obtained?—*O.C.* 1925.

18. Describe, with the aid of diagrams, two common defects of vision.

A short-sighted person can only see objects distinctly if they lie between 7 cms. and 100 cms. from the eye. What kind of spectacles would he require to enable him to see distant objects distinctly?

With these spectacles what would be the least distance of distinct vision?—*Camb.* 1923.

19. Explain the use of a convex lens as a magnifying glass. Is a long or a short focus lens more suitable for this purpose? —*Lond.* 1922.

20. Explain the action of a simple magnifying glass.

A person who sees objects most distinctly at a distance of 25 cms., uses a magnifying glass of 5 cms. focal length. Draw a scale diagram, with brief explanation, showing the action of this lens if it is held near the eye.—*Lond.* 1923.

21. Define the principal focus of a convex lens. Explain by a diagram the use of a convex lens as a simple magnifying glass. and point out what it is that determines the magnifying power. —*C.W.B.* 1921.

22. Explain how a convex and a concave lens can be combined so as to act as a telescope.

A convex lens of 12 cms. focal length is set up so as to form an image of a distant object. Where should a concave lens of 5 cms. focal length be placed so that the rays from a point of the object may emerge as a parallel pencil from the combination? Show, by a diagram, the path of the pencil through the system. —*Camb.* 1924.

23. What is the meaning of *magnifying power* of telescopes?

How would you employ two convex lenses of 10 cms. and 50 cms. focus respectively to form a telescope for observing distant objects? What magnifying power would you expect?—*M.L.L.S.B.* 1925.

24. Two convex lenses of focal lengths 5 cms. and 20 cms. are to be combined to form, (a) a telescope, (b) a microscope. Show how you would place them in each case, and calculate the magnification you would get with one of your arrangements.—*Camb.* 1925.

Part III.
SOUND.

Chapter XI.

LONGITUDINAL WAVE MOTION.

172. Sound and Light. A Contrast. Telescopic observations of the sun show that explosions often take place there on such a vast scale that we should certainly be able to hear them if sound could travel through a "vacuum." Light reaches us easily enough. We have here the obvious contrast that light can travel in the absence of a material medium, while sound cannot.

By "material medium" we mean one which has weight.

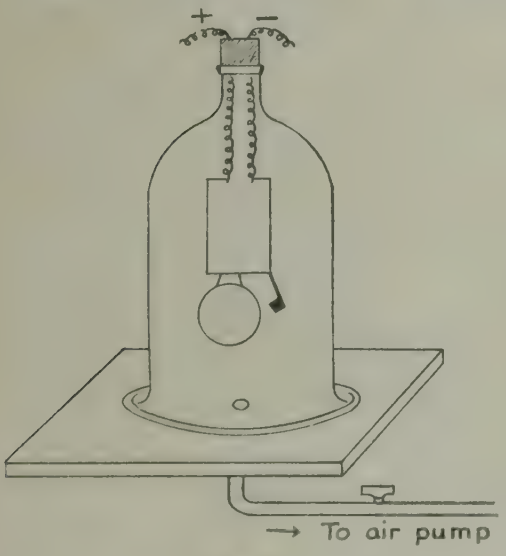

Fig. 131.
Sound cannot travel through a Vacuum.

This necessity for a material medium in the case of sound can be proved by experiment. An electric bell is hung inside the receiver of an air pump. As the air is exhausted the sound becomes fainter and fainter, but becomes louder again as soon as air is admitted.

Before we can advance far in the study of sound, we must understand something about vibration.

173. Vibration. A particle is said to *vibrate* when it repeatedly makes the same (or nearly the same) movement and takes the same (or nearly the same) time in doing so.

The *period of vibration* is the time required for one complete movement.

Note the word "complete." Thus the period of vibration of a pendulum is the time required for the *to-and-fro movement*.

The *frequency of vibration* is the number of complete movements made in one second.

The period is often denoted by t and the frequency by n. Clearly $t = 1/n$.

The *amplitude of vibration* is the distance from the mean position of the vibrating particle to the furthest point reached on one side or the other.

174. How a Sound Wave Travels through Air. Suppose a steel spring, S (e.g., one prong of a tuning fork), moves sharply to the right. What happens?

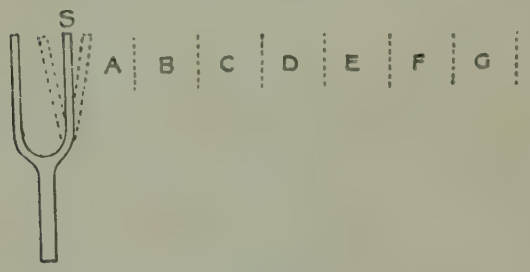

Fig. 132.

Let us imagine the air to be divided into sections, A, B, C . . . . etc. The section A is suddenly compressed, and is also pushed bodily a little to the right. B now responds to A in much the same way as A did to S. It is compressed and moves a little to *its* right, and so on for all the various sections. This successive movement of the portions of air is known as a *pulse*, and it travels with a definite velocity. Thus if, one second after the first movement of S, we find that a "section" of the air 1100 feet away is just experiencing this pressure from the left, we should say that the pulse is travelling with a velocity of 1100 feet per second.

We must distinguish clearly at the outset between the slight bodily movement of the *air* of which we have spoken, and the movement of the *pulse*. The air only moves a very little way (much as the prong of the spring did), while the pulse travels through a great distance.

In the same way, when one sees the wind catch a distant field of wheat, a pulse passes rapidly over the whole field, though each ear of wheat moves only a few inches.

After the prong has moved to the right (with the consequences just described), it will spring back to the left. What will happen now?

Clearly A will make a sudden slight movement to the left, since the pressure on that side has been reduced. Also, as A is occupying a larger volume than it did originally, it will be rarefied. B now experiences less pressure from the rarefied section A than it does from C, hence B will repeat the movement of A. In rapid succession the movement will be repeated by C, D, E . . . . etc. The first pulse of which we spoke is known as a *pulse of compression*, while this second one is known as a *pulse of rarefaction*. The two pulses together make up a complete sound wave. It is important to notice that, in the case of a pulse of compression, the sections of the medium (in this case air) move in the *same direction* as the wave, while in the case of a pulse of rarefaction, they move in the *opposite direction*.

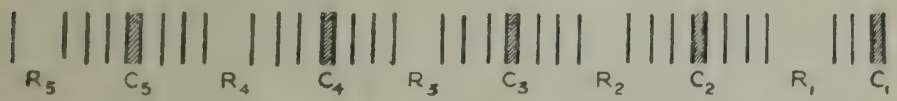

Fig. 133.

Every time the fork moves to the right, a pulse of compression will begin to travel through the surrounding air, and every time it moves to the left a pulse of rarefaction will be set up. Hence we may think of the air as being in a condition represented by fig. 133, certain sections (the C's) being made smaller to indicate that just at that instant they are being compressed, and others (the R's) larger to show that they are being rarefied.

Every movement of the first portion of air is repeated, after definite intervals, by more and more distant portions.

We must now try to be a little more exact. When S begins to move to the right, its movement is at first very

slow (like a pendulum at the beginning of its swing). It soon speeds up, however, reaches maximum speed when in its central position, and then begins to slow down again.

The compression received by A is therefore very slight at first, but it increases in intensity, and after passing through a maximum value gradually declines to zero. Thus the compression passed on is one which shows regularly varying stages of intensity, and similar remarks apply to each rarefaction due to the return journey of the prong. The crudity of Fig. 133 will now be obvious.

Fig. 133 shows sections of compressed air and others of rarefied air, separated by several sections of air at normal pressure. But, owing to the varying speed of the prong of which we have just spoken, the transitions from "compressed" to "normal," and from "normal" to "rarefied," should be gradual and not sudden. Hence the state of things is more correctly represented by fig. 134.

Fig. 134.

Again, unless the air is in a tube, the disturbance in it will become fainter and fainter as the distance from the prong increases.

[Q.—How would fig. 134 have to be modified in order to indicate this "dying away" effect?]

The passage of a sound wave through air is often illustrated by means of the *wave-machine* (consisting in its simplest form of a long coil of stout copper wire, the diameter of the coil being four inches or so). The compressions are indicated by the coils being pushed closer together, and rarefactions by a corresponding opening out.

175. Reflexion of Sound Waves. (See also §§190, 191.) Such a wave-machine is particularly helpful in considering the reflexion of sound waves. Suppose the end Z is fixed, and that a pulse of compression has just arrived there.

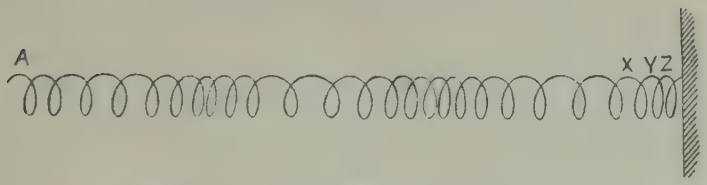

Fig. 135. Simple Form of Wave Machine.

As Z cannot move to the right, it springs back and compresses Y, which in turn compresses X, and so on. Thus a compression is reflected as a compression.

It would take too long to work out the other possible cases, and in fact the student should be able to do this for himself. Here only the results will be stated. At a fixed end, just as a compression is reflected as a compression, so a rarefaction is reflected as a rarefaction. Similarly, when reflexion takes place at a free end, a compression is reflected as a rarefaction and a rarefaction as a compression. We shall have occasion to refer to these results later (§195).

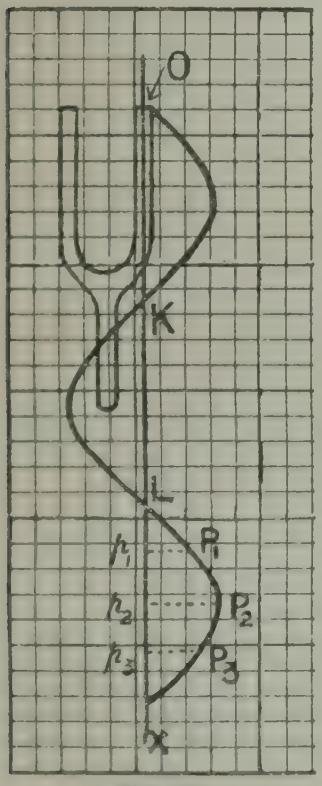

Fig. 136.
Displacement Curve.

176. Displacement Curve. In fig. 136 is represented a strip of squared paper on which is drawn a vertical line, OX. Just in front of this vertical line the prong of a tuning fork is vibrating, and at the tip of the prong we can imagine a very tiny stylo pen. This, of course, would simply record a short horizontal line on the paper. If, however, the paper is now drawn upwards with uniform velocity, the tip of the prong will record a curve such as is shown in the figure, and which is known as a *displacement curve*. Evidently equal distances measured along OX represent equal intervals of time, and in

fact OL represents the time of one vibration. Take any point, P_1, on the curve, and draw P_1p_1 perpendicular to OX. The length, P_1p_1, represents the amount by which the tip of the prong was displaced at the time represented

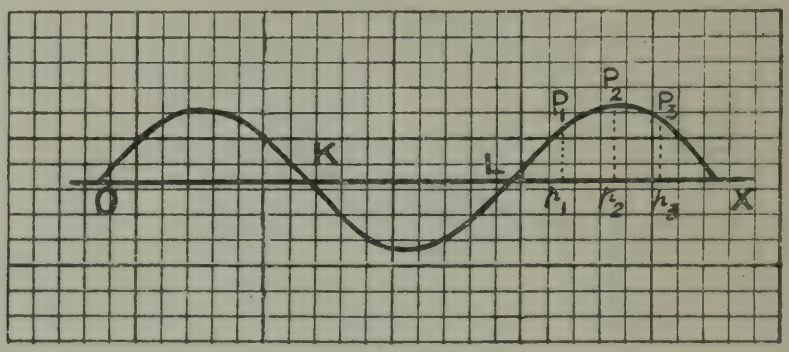

Fig. 137. Displacement Curve.

by the position of p_1. In practice such a curve is usually represented in a horizontal position (fig. 137). A point on the curve above the axis then represents a displacement to the right and vice versa.

177. Application to Sound Wave. Referring again to the effect of the vibrating prong on the neighbouring air, we saw that (fig. 132) the movement of the prong to the right was followed a moment later by a similar movement of A, which was in turn followed by a similar movement of B, and so on. Hence at any given point in the neighbourhood of the prong, the air goes through a cycle of movements represented by the displacement curve.

Next suppose that at a given instant the displacement of a certain section of the air, say G, is represented by the point P_1 in fig. 137. F has already passed through this condition, and is now in a more advanced one represented say by P_2. Similarly the displacement of E may be represented by P_3, and so on. Hence at any given instant the displacement of successive portions of air in the neighbourhood of the prong is represented by the displacement curve.

This curve then represents the displacement both of *successive portions of the air at the same time*, and also of *any given portion of the air at successive intervals of time.*

178. Phase and Wave-length. Suppose a particle is vibrating along a path, AB, its period of vibration (i.e., the time for the complete to-and-fro journey) being t

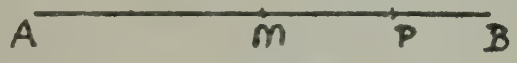

Fig. 138.

seconds. Let M be the middle point of the path. Suppose the particle is in a position P, and that t' seconds have elapsed since it last passed M in the positive direction (i.e., moving from left to right). Then the **phase** of the vibrating particle is represented by the fraction t'/t.

Now consider a succession of particles of air in the neighbourhood of a vibrating body.

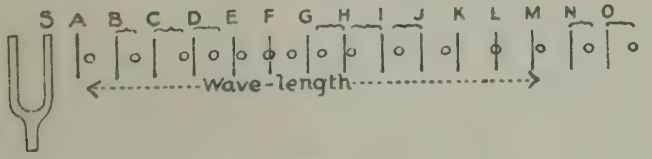

Fig. 139.

S can stand for the vibrating body, and A, B, C, D . . . . for the successive particles. The positions of these particles when at rest are shown by the short vertical lines equidistant from one another. When vibrating, the positions at a given moment are shown by the small circles. B is evidently in a later phase of vibration than A, C than B, and so on, and finally we come to a point which is a whole period behind A, and is therefore in the same phase. If M is this point, then the distance AM is known as the ***wave-length,*** which may thus be defined as **the distance between two consecutive particles which are in the same phase of vibration.**

Suppose n is the frequency of the vibrating body, λ the wave-length, and v the velocity with which the

wave motion advances. After $1/n$th second the body will be in the same phase as at first, and a point somewhere in the air will just be commencing a similar movement, i.e., this point will be in the same phase as S, and the distance from S to the point is one wave-length $=\lambda$. After n vibrations (performed in one second) the disturbance will have travelled n times as far, i.e., to a distance $n\lambda$. But in one second the wave motion advances a distance v. Hence $v = n\lambda$.

179. Musical Sounds and Noises. A musical sound has definite pitch, while a noise has not. A noise is produced by a source in non-periodic motion, giving rise to a disturbance in the air of no definite wave-length, while a musical sound is produced by a source vibrating regularly and producing a regular sequence of similar waves.

180. Loudness, Pitch and Quality. Musical sounds differ from one another in :—

(1.) **Loudness.** (2.) **Pitch.** (3.) **Quality.**

(1.) *Loudness* is connected with amplitude of vibration. Thus a tuning-fork which has been struck hard gives a louder note than one which has been struck gently, the prong having a greater amplitude of vibration in the former case.

(2) *Pitch* is connected with frequency. As the frequency is increased, the pitch rises (§181).

(3.) *Quality*. A note sounded on the violin might be of the same pitch, and the same loudness, as one sounded on the organ, and yet we can distinguish them. They are said to differ in *quality*. Quality is connected with the shape of the displacement curve (§176). Another way of putting it is to say that quality is connected with the number and the intensity of the overtones present (§205).

181. Pitch and Frequency. It was stated in the last paragraph that as the frequency of vibration is increased, the pitch rises. We shall now consider how this may be shown by experiment. Several toothed wheels are

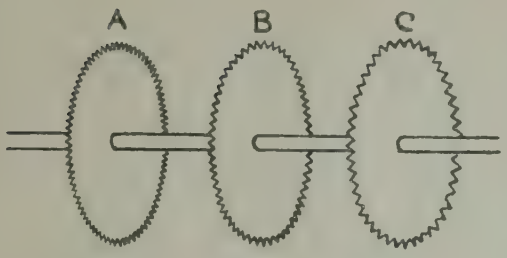

Fig. 140. Savart's Wheel.

mounted on the same spindle, which may conveniently be turned by a small electric motor. The wheels have different numbers of teeth. On turning the spindle and holding a card against the teeth of one of the wheels, say A, a succession of taps is heard at first, which becomes a definite musical note as the speed is increased. With further increase of speed, the pitch of the note rises. The taps correspond roughly to vibrations, and so it is proved that the greater the frequency of vibration, the higher is the pitch.

If B has twice as many teeth round its circumference as A, it will be found that if one card is held against B at the same time as another is held against A, the note from B will be the *octave* of that from A. The apparatus is known as "Savart's Wheel."

More exact results may be obtained by means of the Disc Siren (fig. 141). Air is blown into a small wind-chest, A, the top of which is drilled with a number of holes equidistant from one another and lying on the circumference of a circle. Only one of these, B, is seen in the figure.

Over the top is a disc bored with a corresponding set of holes, the disc being arranged to rotate in a horizontal plane, which only just clears the top of the chest. The holes are not bored vertically, but in the manner indicated in the figure, so that air escaping from the wind-chest has a driving action on the disc. A counting

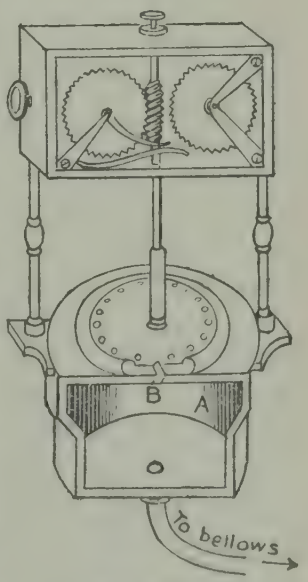

Fig. 141.
The Disc Siren.

arrangement is attached by which the number of revolutions can be recorded.

Suppose we wish to determine the pitch of, say, an organ pipe. We operate the siren until it gives a note whose pitch corresponds to that of the organ pipe. We then keep it moving at the same steady speed so as to maintain the required note, and observe the number of revolutions made during a definite interval. Thus suppose the disc had made 480 revolutions in 20 seconds, i.e., 24 revolutions per second. If the number of holes were 16, there must have been 24×16 or 384 puffs produced per second. Thus the note of the siren (and therefore of the organ pipe) was of frequency 384.

182. Doppler's Principle. If an engine is whistling as it approaches a station, an observer on the platform notices that the pitch rises as the engine approaches him, and falls after it has passed. This peculiarity is known as the *Doppler Effect.*

Explanation. Suppose the frequency of the note is 550 per second, and the velocity of sound is 1100 feet per second. From $v = n\lambda$, we see that for a stationary engine the wave-length λ $(= v \div n)$ must be 2 feet. Now if the engine is approaching at 60 miles an hour $(= 88$ feet per second), it will advance $\frac{88}{550}$ or $\frac{4}{25}$ feet in $\frac{1}{550}$ second, and instead of the wave length being the full 2 feet, it will be 2 feet minus $\frac{4}{25}$ feet $= 1\frac{21}{25}$ feet. Waves of this length reach the observer, and the frequency to him $(= v/\lambda)$ is $1100 \div 1\frac{21}{25} = 598$. Similar reasoning shows that after the engine has passed, the wave-length will be 2 *plus* $\frac{4}{25}$ feet, and the frequency will be $1100 \div 2\frac{4}{25} = 509$.

Since pitch depends on frequency, the pitch of the note from the approaching engine will be definitely higher than that from the receding one.

183. Velocity of Sound in Air. This may be determined in several ways, including :—

(1.) An open-air method (see below).

(2.) The method of Resonance (§197).

184. Open-air Method. A man fires a gun at a known distance from an observer, who by means of a stop-watch records the time at which he sees the flash and that at which he hears the report. Thus if the distance were 5500 feet and the interval 5 seconds, the velocity would be $5500 \div 5 = 1100$ feet per second.

N.B. (i.) The effect of wind must be allowed for. This is done by having two observers at two stations, A and B, and observations are made both for the direction A to B and the direction B to A. By taking the average, the effect of wind is eliminated.

(ii.) The temperature and humidity (i.e., amount of water vapour) of the air must be observed, and corrections applied accordingly (see next paragraph).

185. Conditions affecting Velocity of Sound. It can be shown that if P is the pressure of a gas, D its density, and V the velocity of sound, then $V = \sqrt{\frac{\gamma P}{D}}$ where γ is a constant which for most cases, including air, is 1.41. Hence for air, $V = \sqrt{\frac{1.41\ P}{D}}$

V, P and D must be expressed in suitable units. Thus V may be expressed in feet per second, P in poundals per square foot, and D in pounds per cubic foot.

It is worth while thinking why the velocity should be connected with pressure and density.

If we consider how a sound wave travels through air (§174), we see that it travels because air that has been compressed tends to spring back, i.e., because air possesses *elasticity*. But this elasticity must be closely connected with pressure ; thus the inner tube of a bicycle tyre that has a good pressure of air in it springs back much faster, after being squeezed and then released, than if there is only very little pressure of air. Hence the velocity of sound, since it depends on elasticity, should also depend on pressure ; and this accounts for the presence of P in the formula.

We should expect density (D) to come in too, because a dense medium would not spring back so quickly as a less dense one, just as a heavy man is not usually so nimble as a lighter one. Thus the greater the density, the less the velocity, which is expressed by D being in the denominator of the expression given above.

186. Effect of Pressure. We cannot increase or decrease the *pressure* of a gas (at constant temperature) without increasing or decreasing its *density* in proportion. Since $V = \sqrt{\frac{1.41\ P}{D}}$ and any change in P is "cancelled out" by a corresponding change in D, it follows that *the velocity is independent of the pressure.*

187. Effect of Temperature. As the temperature of a gas increases (at constant pressure) its density decreases, the density in fact being inversely proportional to the absolute temperature.

Referring to the above formula, it follows that *the velocity is proportional to the square root of the absolute temperature.*

188. Effect of Humidity. The action of the barometer reminds us that an increase in the amount of water vapour in the air results in a decrease of density. Hence *the velocity will increase as the amount of water vapour increases.*

When a band is playing in the distance, the low notes and the high notes (i.e., those of longer and shorter wave-lengths respectively) must reach our ears at the same time, otherwise there would be hopeless discord. Hence *velocity is independent of wave-length.* Similarly loud notes and soft ones reach us at the same time, showing that *velocity is also independent of amplitude.*

189. Summarising, we may say that **the velocity of sound in air :—**

(1.) **Is proportional to the square root of the absolute temperature ;**

(2.) **Increases with the presence of water vapour ;**

(3.) **Is independent of the pressure ;**

(4.) **Is also independent of the length of the sound waves and of their amplitude.**

Example 45. If the velocity of sound in dry air at N.T.P., (i.e., 0° C. and 76 cms. pressure) is 1,090 ft. per sec., what would be its velocity at 20° C. and 77.5 cms. pressure?

The change of pressure may be ignored.

0° C. = 273° Absolute, and 20° C. = 293° Absolute.

Call the corresponding velocities V_1 and V_2.

Then $\frac{V_1}{V_2} = \frac{\sqrt{273}}{\sqrt{293}}$

$\therefore V_2 = \frac{V_1 \times \sqrt{293}}{\sqrt{273}} = \frac{1090 \times \sqrt{293}}{\sqrt{273}}$ which works out to **1129 ft. per sec.**

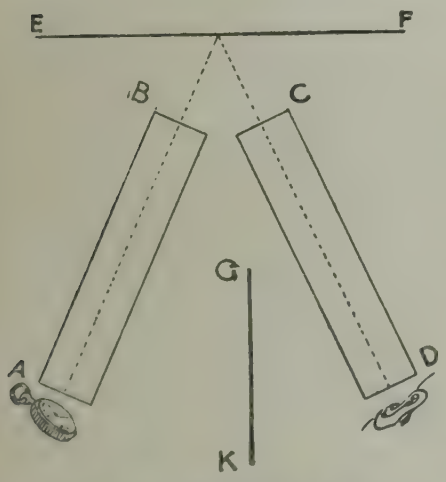

Fig. 142. Reflexion of Sound.

190. Reflexion of Sound. (See also §175.) Two metal tubes, about a yard long and two inches in diameter, are mounted horizontally in front of a board, EF. A watch is suspended near A (fig. 142), and the tube CD is moved round until an ear placed at D can hear it more distinctly than in any other position. GK is a felt screen to prevent sound waves from A reaching D directly. It will be found that the best position is when the two tubes are equally inclined to the normal to the reflecting surface, and in the same plane with it. Hence *sound is reflected according to the same laws as light.*

191. Echoes. Suppose we hear a short sharp sound of practically no duration, the impression is retained for about 1/10th second after the actual sound has ceased. (Cp. §156.) Hence if a second sound followed the first at an interval of less than 1/10th second, we should hear the two sounds as one. This point is important in connection with echoes.

If a boy at a distance of 55 feet from a cliff surface called out a short syllable, say "Bob," the sound would travel to the cliff, and would then be reflected, reaching the boy after making a double journey of 110 feet. As sound travels at 1100 feet per second, this journey would occupy 1/10th second, and so the boy would just be able to hear the echo as a distinct syllable. If he were nearer the cliff, he would not hear the echo as a distinct syllable, because the echo would reach him less than 1/10th second after the original sound, and so the two sounds would run into one.

Example 46. A steamer is in a fog in the neighbourhood of an iceberg. The echo of the whistle is heard $3\frac{1}{2}$ secs. after the production of the original sound. How far is the steamer from the iceberg?

Assume that the velocity of sound is 1100 ft./sec.
In $3\frac{1}{2}$ secs. sound would travel $1100 \times 3\frac{1}{2}$ ft. $= 3850$ ft.
$\therefore$ 3850 ft. is the distance to the iceberg and back again.

$\therefore$ **required distance** $= \frac{3850}{2} =$ **1925 ft.**

[Q.—How could the captain of the steamer find out if he were moving *towards* the iceberg or not?]

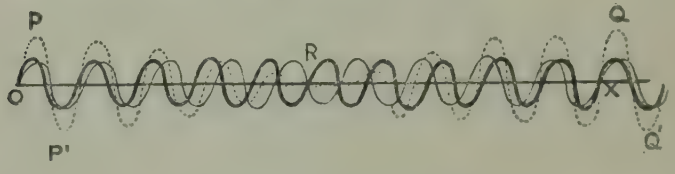

Fig. 143. Illustrating Beats.

192. Beats Suppose two tuning-forks, A and B, have frequencies of 90 and 100 respectively. In fig. 143 the thin line represents the displacements in the air due to A, and the thick line those due to B. Let the "time" line, OX, represent 1/10th of a second, so that in this period 9 vibrations will have been made by A and 10 by B. Let us take it that A and B both begin the period by producing compressions. The effects are added together, so that at first the ear receives an extra strong compression, followed at once by an extra strong rarefaction (see P and P′ in fig. 143), and loud sound is heard. 1/10th of a second later each fork will have completed

an exact number of vibrations, and so the effect (i.e., extra strong compressions and rarefactions resulting in loud sound) will be repeated, as indicated at Q and Q'. At the middle of the period considered, however (see point R), the forks will be vibrating in opposite directions, and the compression effect due to one will cancel out the rarefaction effect due to the other. This is indicated in the figure by the fact that the dotted line, which is the resultant of the two displacement curves, coincides with the line OX. Thus there is a moment of silence coming between two moments of maximum loudness.

In each period of 1/10th second this state of things will recur, and in a complete second there will have been ten moments of silence coming between moments of loudness. These alternations of loudness and silence are known as **beats,** and it will be seen that *the frequency of the beats is equal to the difference between the frequencies of the two sounding bodies.*

The particular frequencies, 90 and 100, were only chosen, of course, for the sake of illustration. Other numbers might have been used.

Beats can be heard as a curious throbbing effect when two sources of sound are vibrating together, provided they do not differ very much in frequency.

It should be added that beats are the result of a particular case of what is known as *interference*, i.e., the principle that if two (or more) systems of waves are travelling through a given medium, each produces the same effect on the medium as if no other system were present.

193. Determination of Frequency. If two tuning-forks, A and B, are of nearly equal frequency, and we know that of A, we can use the method of beats to find that of B. Suppose the frequency of A is known to be 284, and that when sounded with B we count 24 beats in 6 seconds, i.e., 4 per second. Then the difference in the frequencies is 4, so that B is either 288 or 280. Now load B with a little wax, lead wire, etc., so reducing its

frequency. If its frequency were already lower than A's, the number of beats will evidently be increased; otherwise it will be diminished. Hence we can tell which of the two values (288 and 280) is to be selected.

194. Resonance. Think of a plank across a stream. If we give a slight jump when we are near the middle of it, we find that it has a certain natural period of vibration. If in walking across it we time our steps so as to agree with this natural period, very strong vibrations may be set up, and in fact the consequences may be uncomfortable. We have here a rough illustration of ***resonance*** which means **the production of strong vibration in a body by so timing a succession of impulses that the interval between them coincides with the natural period of vibration of the body.**

If two tuning-forks, A and B, are of the same frequency, and A is sounded strongly near B and then suddenly damped, we may hear B continuing to give out the same note. The pulses from A taken singly would have been much too feeble to cause movements in the prongs of B, but a regular succession of impulses, arriving at intervals agreeing with B's natural period of vibration, produces the effect noted.

195. Resonance in a Closed Tube. AB is a glass tube connected by rubber tubing to a small reservoir, R, filled with water. R is supported by the ring of a retort stand, and by moving this ring up or down we can readily alter the level, C, of the water in AB. Thus AC is an air-column, of which the length can very easily be adjusted.

If a vibrating tuning-fork, T, is held over A, we shall probably hear very little sound, but on repeating the experiment a number of times, each time changing the length of the air-column, we find that for one particular length, resonance is produced—a very marked increase in the sound. Why is this?

Suppose T, by moving from *b* to *a*, has just sent a pulse of compression down the tube. At C this is reflected as a compression, which travels back to A

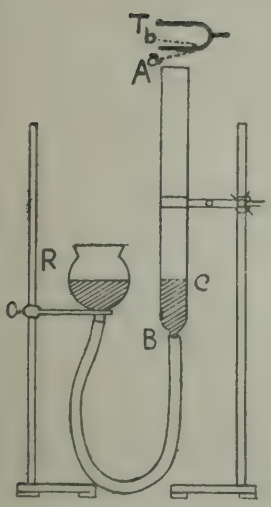

Fig. 144.
Resonance in Closed Tube.

(§175). As A is a "free" end, the compression is reflected as a rarefaction which proceeds to travel down the tube. Now if at the moment that this rarefaction is beginning its journey, the prong is just about to move from *a* to *b*, it is evident that the rarefaction travelling down the tube will be reinforced by the rarefaction produced by the fork.

Following the process a little further, the rarefaction which travels down the tube is reflected at C as a rarefaction, returns to the free end A, and is there reflected as a compression. But by this time the fork will have completed another half-vibration, and will be about to move from *b* to *a*. Hence the compression that would be travelling down the tube in any case, is reinforced by a new compression produced by the fork.

The original pulse will therefore have travelled four times the length of the tube in the time required by the fork to make one complete vibration. This then is the simplest condition for resonance in the case of a tube closed at one end.

Let us now go back to the point at which the pulse had travelled two lengths of the tube, and was just about to return as a rarefaction at the same time that the fork was producing a rarefaction (i.e., half-a-period from the starting point). We should have obtained resonance equally well if the pulse had made the to-and-fro journey in $1\frac{1}{2}$-periods, or $2\frac{1}{2}$, $3\frac{1}{2}$, etc., instead of the half-period. If l is the length of the tube in the first case (half-period) the lengths in the other cases would evidently be $3\,l$, $5\,l$, etc.

In the case of the *shortest* air-column that will resound to a given fork, the wave-length must be four times the length of the air-column, because that is the distance which a pulse travels in the time of one vibration.

After reading §203, it will be seen that we might arrive at this result in another way. Since waves are passing along the tube in both directions, the air inside it is in a state of stationary undulation, the closed end of the tube being a node and the open end an antinode The wave length is four times the distance from node to antinode, i.e. 4 times the length of the air column.

196. Resonance in an Open Tube. In this case the water-level method of varying the length of the tube is evidently unsuitable. We can, however, slide the glass tube into a slightly wider one made of cardboard, which can be slid up and down telescope fashion.

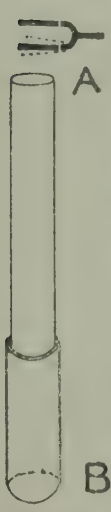

Fig. 145. Resonance in an Open Tube.

Suppose a compression has just started down the tube. At B (fig. 145), a free end, it is reflected as a rarefaction which travels back to A (also a free end), where it is reflected as a compression. The condition for resonance is that the fork should just be producing a compression at the same moment, i.e., *the pulse must have travelled twice the length of the tube in the time of one complete vibration.*

Evidently we should also obtain resonance if the fork had made two, three, etc., vibrations. Therefore if l is the length of the *shortest* air-column that will give resonance, we should also obtain the effect with lengths $2\,l$, $3\,l$, etc.

Since in the simplest case a pulse travels twice the length of the tube in the time of one vibration, the wave length is evidently $2\,l$.

Here again we have a case of stationary undulation, with an antinode at each end and a node in the middle. Since distance from node to antinode $= \frac{1}{2}\,l$, the wave length $= 4 \times \frac{1}{2}\,l = 2\,l$ as before.

The way in which the air is actually moving in different parts of a resounding column will be understood after §203 has been read.

197. Velocity of Sound by Method of Resonance. A fork of known frequency is sounded over a tube (usually closed at one end), and the length of the air column is adjusted until resonance is obtained. This length is then

measured and the velocity is easily calculated. Suppose, for instance, the frequency of the fork is 256, and the shortest column which gives resonance (closed tube) is 12.9 inches. The wave-length is $4 \times 12.9 = 51.6$ inches.

$\therefore v = n\lambda = 256 \times 51.6$ inches $= 1101$ feet per second.

Conversely, if the velocity of sound is assumed to be known, we can determine the frequency of a given tuning-fork.

(N.B. The length of the resounding column is actually a little greater than the length of the tube, and in very exact work it is usual to assume that the *effective* length of the tube is greater than the real length by $0.3\,d$, where d = diameter of tube.

Thus suppose a tube is 80 cms. long and 2 cms. in diameter, we should count the effective length as $80 + .3 \times 2$ or 80.6 cms.

198. Effect of Change of Temperature. Suppose resonance has been obtained with a tube at say 10°, and we then raise the temperature to 20°. At the higher temperature a pulse will travel faster, and so will do its double journey before the fork has completed a half-vibration (closed tube). Hence for perfect resonance the tube would have to be made a little longer.

Similarly we should need a longer tube if we used a light gas (coal-gas, hydrogen, etc.) than if we used air; and a shorter tube if we employed a heavy gas such as carbon dioxide.

QUESTIONS ON CHAPTER XI.

1 Describe an experiment to illustrate the fact that a material medium is necessary for the transmission of sound. Explain how the air moves when sound travels through it.—*Lond.* 1921.

2. Explain the method of representing longitudinal waves by a curved line.—*Lond.* 1924.

3. Write a short account of the production and propagation of sound in air.—*Lond.* 1925.

4. Define *transverse* and *longitudinal* wave motion, and give an example of each.

Define *wave length* and *frequency*, and show that the product of wave-length and frequency is equal to velocity of propagation. —*Oxf.* 1924.

5. Explain the terms *frequency*, *amplitude* and *velocity*, as applied to sound waves in air, and state how the sensation of

sound is altered by variations in these quantities. The human ear normally responds to sound waves with wave-lengths ranging from 2 cms. to 1000 cms. Determine approximately the number of octaves represented by this range. (Velocity of sound in air, 33,000 cms. per sec.)—*Camb.* 1923.

6. On what does the intensity of a sound depend? Explain the production of loud and soft sounds by the human voice.

What is the difference between sound waves which produce a harsh effect and those which produce a pleasing effect on the ear? —*Oxf.* 1924.

7. Distinguish between the pitch, the intensity and the quality (or timbre) of a musical note; and say on what physical fact each depends. How can, (a) the pitch, (b) the intensity, of a given tuning-fork be varied? Give an explanation in each case. —*Scot.L.C.* 1927.

8. Define amplitude and wave-length.

How would you determine the wave-length in air of the note emitted by a tuning-fork?—*Lond.* 1921.

9. Describe how to show, (a) that the frequency of a note is doubled when its pitch is raised an octave, (b) that fixing a small piece of wax on the prong of a tuning-fork lowers its pitch.—*Lond.* 1922.

10. Describe an experiment for showing a relation between the pitch of a note and the frequency of the vibration which causes the note.—*Lond.* 1926.

11. When an express train is approaching a station, the pitch of the engine's whistle appears to a person standing on the platform to be higher than when the train is receding. How is this explained? If the explanation you give is the correct one, how should the difference in pitch vary with, (a) the speed of the train, (b) the velocity of sound (as affected by temperature)?—*Scot.L.C.* 1926.

12. Explain the terms frequency, wave-length, and velocity, as applied to sound waves.

Describe an experimental method of determining the velocity of sound in air at atmospheric pressure.—*Camb.* 1924.

13. Do sounds of all kinds travel with the same velocity in air? What conditions in the air itself may affect the velocity? Give experimental evidence for your answer.—*Lond.* 1926.

14. How can the velocity of sound in air be determined? What effect, if any, is produced on this velocity by, (a) a rise of temperature, (b) a rise in the barometric pressure? Give a reason in each case.—*Scot.L.C.* 1926.

15. How does the velocity of sound in air depend upon, (1) temperature, (2) pressure?

A boy standing in a disused quarry claps his hands sharply, once every second, and hears an echo from the face of an opposite

cutting. He moves until the echo is heard mid-way between the claps. How far is he then from the reflecting surface, if the velocity of sound at the time was 1120 ft. per sec. ?—*Lond.* 1923.

16. Explain how echoes are produced. If a man 256 metres away from a large building fires a pistol and hears an echo $1\frac{1}{2}$ secs. after the report, what is the velocity of sound in the air ? How would the velocity be affected by changes in the pressure and temperature of the atmosphere ?—*Camb.* 1921.

17. Explain the production of beats.

Two tuning-forks, one of which has a frequency of 256, give four beats per sec. when sounded together. What are the possible values for the frequency of the other fork, and what experiments would you make to determine which of these values is the actual frequency of the fork ?—*Camb.* 1924.

18. Give an explanation of each of the following phenomena : (a) The pitch of a piece of music played on a gramophone may be varied by varying the speed at which it is played. (b) With a closed organ pipe, the lowest note that can be produced has its wave length four times the length of the pipe. (c) If two adjacent black and white keys of a piano are struck simultaneously, an unpleasing effect is produced.—*Scot.L.C.* 1927.

19. What do you understand by resonance ?

Give an account of any experiments on resonance with which you are familiar, and explain the purposes for which they are used.—*Camb.* 1921.

20. How may the velocity of sound in air be measured by means of an organ pipe of adjustable length and a tuning-fork ? Describe the motion of the air in the pipe when the length is adjusted to resonance.—*Camb.* 1924.

21. Upon what does the frequency of the note emitted by an organ pipe depend ?

If the frequency of the note emitted by an organ pipe is 260 in a room at a temperature of 0° C., what will its frequency be if the temperature rises to 27° C. ?—*Camb.* 1925.

22. When a vibrating fork is held over the open end of a tube, the other end of which is closed, the sound is often heard more loudly than when the fork is not over the tube. Under what conditions can this occur ? How may experiments of this kind be used to find the wave-length of the note emitted by the fork ?—*Oxf.* 1924.

23. A tube, 3 cms. in diameter and 50.5 cms. long, open at both ends, resounds to a fork giving 320 vibrations per second. Explain, with the help of a diagram, how the air is vibrating in the tube, and calculate the velocity of sound in the air.

When the tube is kept full of coal gas, it has to be made 11.5 cms. longer in order to resound to the same fork. Why is this ?—*Camb.* 1923.

Chapter XII.

TRANSVERSE WAVE MOTION.

199. Transverse Waves. In the case of the waves we have considered so far, the vibrating material has always moved, backwards or forwards, in the line along which the wave is travelling. Such a wave is said to be *longitudinal*. If the vibrating material moves at right angles to the direction of the wave, the wave is said to be *transverse*. Thus a wave passing across the surface of a pond is transverse, for the water bobs up and down at right angles to the direction in which the wave is travelling.

200. How a Transverse Wave Travels. Suppose A, B, C, D . . . . are successive points along a stretched wire, and that A is plucked upwards and then released. It can be shown that A will vibrate up and down. How will B, C and the succeeding particles be affected?

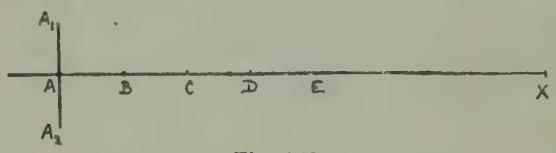

Fig. 146.

When A moves upwards to A_1, the string obviously slants upwards from B to A_1, and so B is pulled up. B in turn is followed by C, and so all along the line. Similarly the downward movement of A will be followed by a corresponding movement of B, which in turn will pull C downwards, and so on.

Thus the movement of A is repeated, at later and later intervals, by similar movements of B, C, etc. When A has completed one vibration (i.e., has moved over the path, $A_1AA_2AA_1$), there will be some point, X, which is just beginning to repeat the first upward movement of A. Thus A and X will be in the same *phase*, and the distance, AX, will be the *wave-length* ("the distance between two consecutive particles in the same phase," §178). The distance, AA_1 or AA_2, will be the *amplitude*, and the number of complete vibrations made per second by A (or by any of the succeeding particles) will be the *frequency*.

When A had completed one vibration, the wave had travelled to X, a distance λ (where λ is the wave-length.)

If A makes n vibrations in a second (i.e., if n is its frequency), then after one second the wave will have travelled to a distance of $n\lambda$. But the distance travelled in one second is the velocity. Hence $v = n\lambda$ (cp. §178).

It will be convenient to notice here that the velocity of the wave, (i.) increases as the tension of the wire increases (for with increased tension B, C, D, etc., will move upwards, and afterwards downwards, more rapidly), (ii.) decreases as the mass per unit length of the wire increases (for a greater mass, acted on by a given force, will not move so rapidly as a smaller mass). It can be shown in fact that $V = \sqrt{\frac{T}{m}}$ where :—

V = velocity of wave,
T = tension (or stretching force),
m = mass per unit length.

This formula should be compared with the formula for the velocity of sound in air. T is the force which makes a displaced portion of the wire spring back after displacement ; P (the pressure) is the corresponding force in the case of the air.

201. Displacement Curve. In §176 we considered the displacement curve in the case of a longitudinal wave, and we saw that when a point on the curve was above the axis, it corresponded to a displacement to the right, and vice versa.

The displacement curve in the case of a transverse wave is quite similar. Here, however, the distance above or below the axis corresponds to the upward or downward displacement of the wire or other material. In fact the shape of the curve may represent the actual shape of the wave.

As in the case of a longitudinal wave, the displacement curve represents both :—

(i.) The position of a given point in the medium at successive instants, and

(ii.) The position of various points in the medium at the same instant.

202. Reflexion of a Transverse Wave. Suppose a transverse wave travelling from A to Z meets an obstacle

Fig. 147.

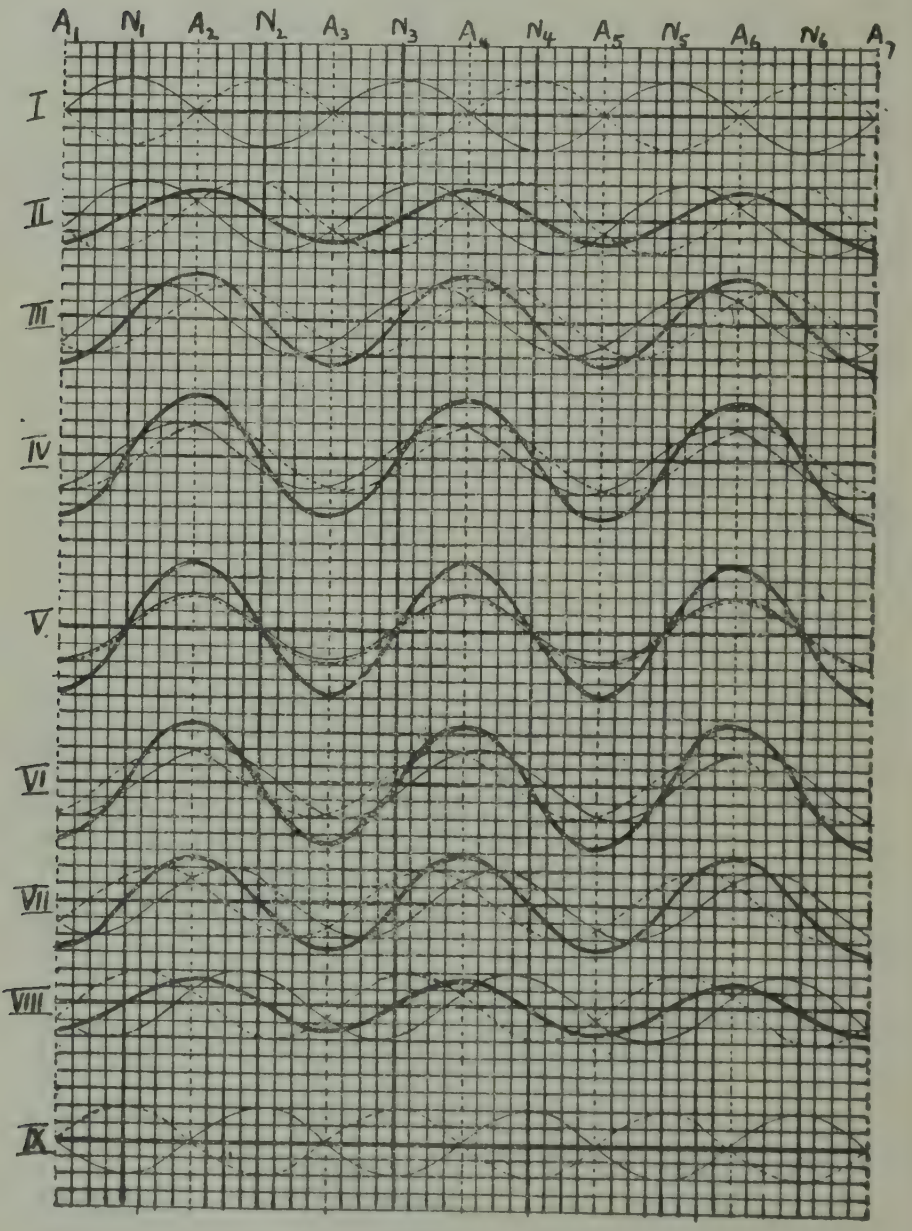

Fig. 147 (a). Stationary Undulation.

at Z (AZ might be a wire, for instance, clamped at Z). As Z is a fixed point, the part ZP will spring back into the position ZP′, and we have the beginning of a reflected wave.

We now have an incident wave travelling from left to right, and a reflected wave travelling in the opposite direction.

Let us try to trace out the results of the *interference* (§192) of these two waves. In fig. 147 (a), I.—IX., the incident wave is shown advancing by successive stages, each stage being one-sixteenth of a period later than the preceding one. Hence from I. to IX. we have the same displacement curve, but each curve is further to the right, by one-sixteenth of a wave-length, than the one before it.

But, as the incident wave moves to the right, the reflected wave moves at an equal rate to the left. Thus we are able to show the positions of the two waves at successive intervals, and from these two curves we can easily obtain the displacement curve which is the resultant of the two original ones.

The incident wave is represented by a thin line and the reflected wave by a dotted one; the resultant wave (i.e., the one formed by combining the other two) is represented by a thick line.

This curve shows some interesting peculiarities, of which we shall select three.

(i.) At certain points, N_1, N_2, N_3 . . . . the displacement is always zero. These points are called *nodes*.

(ii.) At other points, A_1, A_2, A_3 . . . . situated exactly half-way between the nodes, the displacement is always a maximum. These points are called *antinodes*.

(iii.) The distance between two successive nodes, or between two successive antinodes, is equal to half the length of the incident (or reflected) wave. From a node to the adjacent antinode is a quarter of a wave-length.

The peculiar type of wave-motion just considered is known as ***stationary undulation***.

203. Reflexion of Longitudinal Waves. It is convenient to notice here that stationary undulation may be set up by the reflexion of a longitudinal wave. This in fact is the state of things which exists in a resounding column of air (§§195, 196). The curves already drawn will illustrate the process, but in this case distance above (or below) the axis represents the degree of displacement to the right (or left) of the air at that particular point.

We have points, N_1, N_2, N_3 . . . . called nodes, where the displacement to the right or left is always zero—in other words, the nodes are points of zero amplitude ; and points, A_1, A_2, A_3 . . . . called antinodes, which are points where the amplitude of vibration is a maximum.

204. Sound Waves and Light Waves Compared. Now that some explanation of transverse wave-motion has been given, we are in a position to make a short comparison of the propagation of sound waves with that of light waves.

Sound Waves.	Light Waves.
Comparatively long and of low frequency.	Very short, and of high frequency.
Comparatively low speed.	Very high speed.
Longitudinal.	Transverse.
Material medium necessary.	Conveyed by the ether.
Both may be reflected, refracted and absorbed.	

205. Modes of Vibration of a String. A string used for musical purposes, a violin string for instance, is fixed at two points which in the following discussion will be referred to as the "ends." Thus one "end" of a violin string is the point at which it passes over the bridge,

and the other is the point at which it touches the screw-peg.

When such a string is plucked or bowed, transverse waves travel to the two ends, and are there reflected. These reflected waves, in passing one another, give rise to the "stationary undulation" just discussed.

The two ends will evidently be points of zero displacement or "nodes," and they *may* be the only ones. If the string is plucked at its middle point, we can make it vibrate in a single "ventral segment" or "loop" (fig. 148a). This is its simplest mode of vibration, and the note it gives out in this case is known as the **fundamental.**

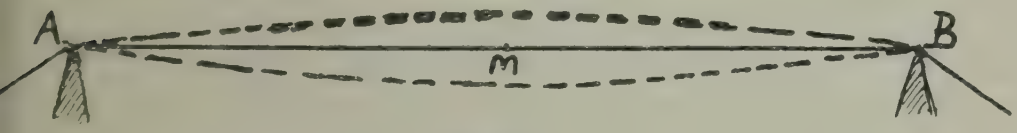

Fig. 148a.

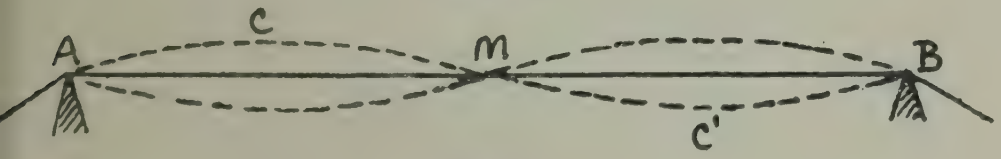

Fig. 148b.

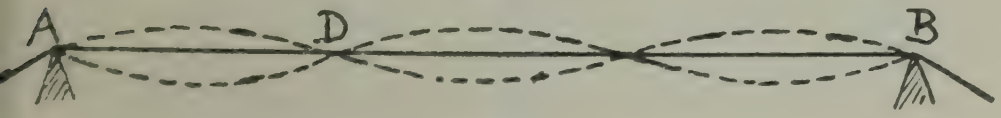

Fig. 148c.

Suppose now we touch the string lightly at M with a feather or rubber pad (so making M a node), and bow the string at C, where AC = $\frac{1}{2}$ AM or $\frac{1}{4}$ AB. The string will now vibrate in two loops (fig. 148b). We may suppose that AM vibrates as a single loop, and at once produces in MB a vibration which is similar but in opposite phase. M acts as a sort of pivot, so that an upward movement of the string on its left causes a downward movement on its right and vice versa.

The string is now said to be sounding its **first harmonic** or **first overtone.** This is an octave above the fundamental. We must pause a moment to consider why.

As will be seen in §208, the frequency of a string vibrating in one segment varies inversely as its length. Thus when a string is vibrating in two segments, each is only half the length of the same string vibrating in one segment, and so the frequency of the first harmonic is twice that of the fundamental. Hence the first harmonic is the octave of the fundamental.

To make the string vibrate in three segments we must make D a node, where AD = 1/3 AB. So we touch the string lightly at D, and bow the string at a point midway between A and D (fig. 148c). We now obtain the second harmonic, and since AD = 1/3 AB, the second harmonic will have a frequency equal to three times that of the fundamental.

In a similar way higher harmonics may be obtained.

In practice it is not easy to make a string vibrate so as to give one particular harmonic to the exclusion of others, but with care we can usually secure that a given harmonic shall be more prominent than the others. The *quality* of a note (§180) depends on the particular overtones present, and on the strength of each.

206. To show the Position of the Nodes. Suppose a wire is vibrating in (say) three segments, and we wish

Fig. 149. Finding Position of Nodes.

to locate the position of the nodes by experiment. We place a number of light paper riders along the wire. Now every point of the wire, *except the nodes*, is undergoing rapid changes of position. Hence the nodes will be those points at which the riders are not thrown off (fig. 149).

207. An Important Formula. It was pointed out in §200 that $V = \sqrt{\frac{T}{m}}$ where V = the velocity of a transverse wave along a stretched string, T = tension, and m = mass per unit length. But we have also that $V = n\lambda$ where λ = length of transverse wave and n = frequency.

$$\therefore\ n\lambda = \sqrt{\frac{T}{m}}$$

$$\therefore\ n = \frac{1}{\lambda}\sqrt{\frac{T}{m}}$$

Again, we saw that $\lambda = 2l$ where l is the distance between successive nodes (§202).

$$\therefore\ n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

For a string vibrating in its simplest mode (i.e., in one loop), the "successive nodes" are at the two ends of the string, and so l is the length of the string.

208. The Laws of Stretched Strings. It follows from the above formula that, **in the case of a string vibrating in its simplest mode, the frequency of the note emitted varies :—**

(i.) **Inversely as the length ;**

(ii.) **Directly as the square root of the stretching force ;**

(iii.) **Inversely as the square root of the mass per unit length.**

To verify the above laws, a *sonometer* is used. It consists of a string, AB, mounted on a sounding board by means of wrest-pins, and of a second string, CD, mounted on the same board, and so arranged that the tension can be readily altered by adjusting the weights at E. The instrument may be used either in a horizontal position or a vertical one, the latter being shown

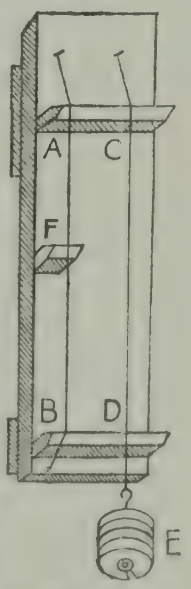

Fig. 150.

in the sketch. The lower part should project a little more than the upper, so that the wire CD may rest against the bridge at D.

209. To show that the Frequency varies Inversely as the Length. Several tuning-forks of known frequencies are obtained, say 256, 384 and 512. The bridge, F, is moved until the string AB (now really FB) is in exact unison with the first fork (256). When F is nearly in the right position, beats will be heard as the string and fork are sounded together. F is moved until these beats just disappear. In making the string give its note, we should pluck it or bow it at about its middle point. [Why ?]

When the string is exactly in unison with the fork, the length, FB, is noted. Suppose this is 90 cms.

F is now moved until the string is in unison with the second fork (384), and then until it is in unison with the third (512), the "effective lengths" of the string being noted in each case. Suppose they are 60 cms. and 45 cms. respectively. We may set out our results thus :—

Frequency.	*Length.*	*Product.*
256	90	23,040
384	60	23,040
512	45	23,040

Since the product is constant, we have an illustration of the law that frequency varies inversely as length.

[Q.—Why "illustration" rather than "proof" ?]

Another good method is available if we assume that the relative frequencies of the notes of the major diatonic scale (doh, ray, mi, fah, soh, lah, te, doh') are known. These ratios may be found by the Disc Siren (§181), and are 24 : 27 : 30 : 32 : 36 : 40 : 45 : 48.

F is adjusted till it is in tune with CD. Call this note "doh," and measure the length FB. Move F until the string gives "ray" (the interval is easily recognised by sounding the two strings in succession). Again measure the length, and proceed in this way for all the notes of the octave. A table is then made out as already indi-

cated, and with careful work the products will be found to be constant within about 1 %.

210. To show that the Frequency varies as the Square Root of the Stretching Force. Using tuning-forks as before, CD is stretched by putting on weights at E until it is in unison with the first fork (256), and the weight is noted. The weights are also determined which bring the string into unison with the second fork (384) and the third (512) respectively. The required weights will be found to be in the ratio $1 : \frac{9}{4} : 4$ or $1^2 : (\frac{3}{2})^2 : 2^2$. As the frequencies are in the ratio $1 : \frac{3}{2} : 2$, we see that the frequency varies as the square root of the stretching force.

Without using tuning-forks, good results may also be obtained by assuming the relative frequencies of the notes of the major diatonic scale, as indicated above, the string AB being employed as a standard "doh."

Or again, we may proceed thus. A given weight is placed at E, and the position of F is adjusted until the two strings are in unison (absence of beats). The length FB is carefully noted. The experiment is repeated for different weights on the one wire, and correspondingly different lengths on the other. Examination of the numbers shows that these lengths vary inversely as the square roots of the stretching force $\left(l \propto \frac{1}{\sqrt{\mathrm{T}}}\right)$. But the lengths themselves vary inversely as the frequencies $\left(l \propto \frac{1}{n}\right)$. Therefore the frequency varies directly as the square root of the stretching force $(n \propto \sqrt{\mathrm{T}})$.

211. To show that the Frequency varies Inversely as the Square Root of Unit Mass of the String. Take any two strings (either of the same or different material) which differ as regards their mass per unit length. This must be determined for each string by weighing a measured length of it. The heavier string is now stretched by a suitable weight, in the position CD, and F is adjusted until this heavier string and the reference string are in unison.

The lighter string is now substituted for the heavier one, the stretching force being the same as before. F is again adjusted until unison is obtained. In each case the effective length of the reference string is carefully measured. It will be found that the effective lengths of the reference string vary *directly* as the square root of unit mass of the string, and therefore the frequency varies *inversely* as the square root of unit mass.

212. To Verify the Formula, $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$ This may be done by adjusting the weights at E until CD is in unison with a tuning-fork of known frequency. m may be found by weighing and measuring either CD itself or a piece of similar wire. Note that T must be expressed in dynes (1 gm. = 981 dynes), m being in gms. per centimetre length, and l in centimetres. All the quantities being known, the formula is then readily verified.

Example 47. What would be the frequency of the fundamental note given out by a string, 80 cms. long, stretched with a force of 5 kilos. 1 metre of the string was found to weigh .19 gms.

Here $l = 80$ cms., $T (= 5$ kgms. $= 5000$ gms.$) = 5000 \times 981$ dynes. $m = .19 \div 100 = .0019$ (gms. per cm.)

$$\therefore\ n = \frac{1}{160}\sqrt{\frac{5000 \times 981}{.0019}}$$

$\log n = \log 1 - \log 160 + \frac{1}{2}(\log 5000 + \log 981 - \log .0019)$.
Working this out, we obtain $n =$ **318.**

Example 48. A string when stretched by a weight of 6 kgms. gives out a certain note. What additional weight will be required in order to make the note rise to the octave above?

Since the note rises to the octave, the frequency is doubled (§181).

Let T_2 kgms. be the tension when the octave is sounding.

Frequency $\propto$ square root of tension (§208).

$$\text{i.e., } \frac{\text{1st frequency}}{\text{2nd frequency}} = \frac{\text{sq. rt. of 1st tension}}{\text{sq. rt. of 2nd tension}}$$

$$\therefore\ \tfrac{1}{2} = \frac{\sqrt{6}}{\sqrt{T_2}}$$

Squaring, $\frac{1}{4} = 6/T_2$, whence $T_2 = 24$.

$\therefore$ the *additional* weight $= 24 - 6 =$ **18 kgms.**

Example 49. A and B are two stretched wires which are being compared with a reference wire, X. A is in unison with 75 cms. of X and B with 100 cms. of X. In what ratio must the tension of B be increased to bring it into unison with A?

Since frequency varies inversely as length (§208).

$\frac{\text{frequency of A}}{\text{frequency of B}} = 100/75 = 4/3.$

But frequency $\propto$ sq. rt. of tension (§208).

$\therefore$ if T_A = tension of A and T_B = tension of B,

We have $\frac{\text{frequency of A}}{\text{frequency of B}} = \frac{\sqrt{T_A}}{\sqrt{T_B}} = \frac{4}{3}$

Squaring, $T_A/T_B = 16/9$; which gives $T_B = \frac{9}{16} T_A$

$\therefore$ tension of B must be increased **in the ratio of 16 : 9** in order to bring it into unison with A.

Example 50. Two stretched strings, A and B, are in unison A's length is now reduced by one-third. How must the tension of B be altered so that it may still be in unison?

A's *length* has been reduced in the ratio 3 : 2.

$\therefore$ its *frequency* has been increased in the ratio 2 : 3.

Suppose the tension of B changes from T_1 to T_2.

Frequency $\propto$ sq. rt. of tension (§208).

$\therefore \frac{2}{3} = \frac{\sqrt{T_1}}{\sqrt{T_2}}$

Squaring, $\frac{T_1}{T_2} = \frac{4}{9}$

$\therefore$ tension of B must be **increased in the ratio 9 : 4.**

Example 51. Under a tension of 12 lbs., a string gives out a certain note. It is then exchanged for a second string of the same material, but of half the length and twice the diameter. What tension will be required if the same note is to be produced?

In this case so many of the quantities change, that it will be better to employ the full formula. For the first string we may write $n_1 = \frac{1}{2l_1}\sqrt{\frac{T_1}{m_1}}$ (i.);

and for the second $n_2 = \frac{1}{2l_2}\sqrt{\frac{T_2}{m_2}}$ (ii.)

There is no change in the frequency, i.e., $n_2 = n_1$.

Also $l_2 = \frac{1}{2} l_1$ (given). Further, if d stands for diameter, $d_2 = 2d_1$. But the mass per unit length is proportional to the square of the diameter, $\therefore m_2 = 4m_1$.

From (i.) and (ii.) above, by division, we have

$$\frac{n_1}{n_2} = \frac{\frac{1}{2l_1}\sqrt{\frac{T_1}{m_1}}}{\frac{1}{2l_2}\sqrt{\frac{T_2}{m_2}}} = \frac{l_2}{l_1}\sqrt{\frac{T_1 m_2}{T_2 m_1}}$$

But $n_1 = n_2$, $l_2 = \frac{1}{2} l_1$, $T_1 = 12$ lbs., $m_2 = 4m_1$ (above).

$$\therefore \frac{n_2}{n_2} = \frac{l_2}{2l_2}\sqrt{\frac{12 m_2}{T_2 \times \frac{1}{4} m_2}}$$

$$\therefore \quad 1 = \tfrac{1}{2}\sqrt{\frac{48}{T_2}}$$

Squaring, $1 = \frac{48}{4T_2}$

Whence $T_2 = \mathbf{12\ lbs.}$

QUESTIONS ON CHAPTER XII.

1. What do you understand by a transverse wave? Illustrate your answer with the aid of a diagram showing the passage of such a wave along a line of equally spaced particles, and explain the terms amplitude, wave-length and frequency.—*Lond.* 1923.

2. Compare and contrast in two parallel columns the chief facts concerning the propagation of light and sound.—*Lond.* 1921.

3. How would you demonstrate the different modes of vibration of a stretched string? In what ratio are the frequencies of these modes?—*Lond.* 1924.

4. How does the note given by a tuning-fork differ from that of a violin string of the same pitch?

If the note given by a string is not quite in tune with that of a fork, by what different means could you bring the two into unison? Explain fully the reasons for your procedure.—*Camb.* 1921.

5. The number of vibrations per second of a stretched string, when it emits its fundamental note, is given by the formula, $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$ Interpret each of the letters in this formula, and describe the experiments by which you tested its truth. In the case of the violin, explain, (a) how a range of notes can be got from a single string, (b) why a change of temperature causes a sharpening or flattening of the fundamental note of a string, (c) why one of the strings is metal wound.—*Scot.L.C.* 1926.

6. Explain the term *overtone*.

Describe how to compare the frequencies of the notes given by two wires of different materials when their lengths and tensions are the same, using a two-string sonometer fitted with a reference wire.—*Lond.* 1921.

7. Describe how to verify the relation between the frequency of a stretched string and its tension, using a series of tuning-forks of known frequencies.

A, B and C are three stretched wires. A is in unison with 80 cms. of C, B with 60 cms. of C. In what ratio must the tension of A be altered to bring it into unison with B ?—*Lond.* 1922.

8. If you were provided with a series of tuning-forks of known frequencies, how would you proceed to verify the relations between the frequency of a stretched string and its length ?

Two tuning-forks, A and B, are in unison with lengths of 50 cms. and 75 cms. respectively of a stretched wire. How would you alter the tension on the wire so that A vibrates in unison with 75 cms. of it ?—*Lond.* 1923.

9. Describe simple methods of altering, (i.) the tension, (ii.) the length, of a sounding wire.

Two such wires are in unison. The tension of one is doubled. How must the length of the other be altered so as again to be in unison with the first ?—*Lond.* 1924.

10. Describe how the frequency of a musical note may be determined.

A string emits a certain note when stretched by a weight of 10 lbs. and plucked. What stretching weight is required for a second string of the same length and material, but of twice the density, which emits the same note an octave lower ?—*Camb* 1922.

11. How does the frequency of the note emitted by a stretched string vary with the length of the string ? Assuming this relation to be known, how would you investigate experimentally the relation between the frequency and the stretching force ? What relation would you expect to find ?—*Camb.* 1926.

12. Describe, in detail, how you would investigate the manner in which the frequency of vibration of a string depends upon the tension to which it is subjected.—*M.L.L.S.B.* 1925.

13. A fine wire is stretched tightly and plucked with the finger. Describe as fully as you can the conditions determining the loudness, the pitch and the quality of the note you would hear.—*Camb.* 1924.

14. Describe the experiments you would make to find out the relation between the length of a stretched string and the frequency of the note it given when plucked or bowed.

A stretched string 1 metre long is divided by two bridges into three parts so as to give the notes of the common chord whose frequencies are in the ratio 4 : 5 : 6. Find the difference between the bridges.—*Lond.* 1927.

APPENDIX I. HEAT.

(a) DEFINITIONS.

Absolute Temperature. Temperature reckoned on a scale of which the **zero** is that temperature at which the volume of a gas would be reduced to nothing at all if it continued to obey Charles' Law.

This **Absolute Zero** is —273° C., and any absolute temperature may be obtained by adding 273 to the corresponding Centigrade temperature.

Boiling Point. The temperature at which the vapour pressure of a liquid becomes equal to that of the atmosphere.

British Thermal Unit. The quantity of heat required to raise the temperature of 1 pound of water through 1 degree Fahrenheit.

Calorie. The quantity of heat required to raise the temperature of 1 gram of water through 1 degree Centigrade.

Capacity for Heat (or **Thermal Capacity**). The number of calories required to raise the temperature of a body by 1 degree Centigrade.

Coefficient of Linear Expansion. Suppose that a solid substance of length x has its temperature raised by 1 degree Centigrade, and that as a result it becomes longer by an amount y. Then the ratio y/x is known as the coefficient of linear expansion of that substance.

"**Coefficient of superficial** (or **surface**) **expansion**" and "**coefficient of cubical expansion**" are similarly defined.

Conduction. A process by which heat passes from a hotter to a colder part of a body, or from a hot body to a colder body in contact with it. There is no transference of the portions of the body lying between the hotter and colder part, but these are all raised in temperature.

Convection. A process by which heat is carried from one part of a liquid or gas to another by the actual transference of the heated portions of the fluid.

Dew Point. The temperature at which the water vapour actually present in the air would be sufficient to saturate it.

Diathermancy. The property of a body in virtue of which it allows heat waves to pass through it.

Efficiency of an Engine. The ratio x/y, where x is the work done by the engine, and y is the mechanical equivalent of the heat with which the engine was supplied in order to do that work.

Energy. Capacity for doing work.

Evaporation. The conversion of a liquid into vapour; it is distinguished from boiling by the fact that it takes place *at any temperature,* and *only at the surface* of the liquid.

Fixed Points of a Thermometer. Points which are marked with reference to two unvarying temperatures, usually the melting point of ice ("lower fixed point") and the boiling point of water under normal atmospheric pressure ("higher fixed point").

Foot-pound. The work done when a force of 1 pound moves its point of application through a distance of 1 foot.

Horse Power. A rate of working of 550 foot-pounds per second.

Humidity, Relative. (See **Relative Humidity.**)

Kinetic Energy. That capacity for doing work which a body possesses in virtue of its motion.

Latent Heat of Ice (or **Latent Heat of Fusion of Ice**). The number of calories required to convert 1 gram of ice at 0 C. into water at the same temperature.

Latent Heat of Steam (or **Latent Heat of Vaporisation of Water**). The number of calories required to convert 1 gram of water at the boiling point into steam at the same temperature.

Mechanical Equivalent of Heat. The number of units of work required to produce 1 unit of heat. Thus 778 foot-pounds would be the mechanical equivalent of 1 British Thermal Unit.

Mist. An effect produced by the condensation of water vapour into minute drops of water which remain

suspended in the atmosphere near the surface of the earth. If the effect is very strongly marked it is known as a **fog.**

Radiation. A process in which heat passes from one place to another in straight lines, with great speed, and without heating the medium through which it passes.

Regelation. The conversion of water into ice following upon the reverse process, both changes being effected by variations of pressure and not of temperature.

Relative Humidity. The ratio of the amount of water vapour actually present in a portion of the atmosphere, to the amount that would be required to saturate it at the existing temperature.

Specific Heat. The specific heat of a substance is the ratio x/y, where y is the quantity of heat required to raise the temperature of a certain mass of water through 1° C., and x is the quantity required to raise the temperature of an equal mass of the given substance through 1° C.

States ("Variable" and "Steady") When two points of a body are kept at different temperatures so that heat is conducted from one to the other, the intervening portion is said to be in the **variable state** so long as the temperature of any point in it is changing, and in the **steady state** when the temperature of every point in it has ceased to change.

Therm. A unit of heat largely employed in the gas industry, and equal to 100,000 British Thermal Units.

Thermal Capacity. (See **Capacity for Heat.**)

Water Equivalent. The water equivalent of a calorimeter (or other piece of apparatus) is the number of grams of water which would be raised 1° C. by the heat required to raise the temperature of the calorimeter, etc., 1° C.

(b) LAWS.

Boyle's Law. If the temperature of a given mass of gas remains constant, then the volume is inversely proportional to the pressure.

Charles' Law. If a given mass of gas is kept at constant pressure, then its volume varies directly as its absolute temperature.

Law Connecting Pressure and Temperature. If a given mass of gas is kept at constant volume, then its pressure varies directly as its absolute temperature.

Newton's Law of Cooling. The rate at which a hot body loses heat is proportional to the difference between its own temperature and that of its enclosure, and does not depend on any other circumstance except the nature and extent of its surface.

(c) FORMULÆ AND OTHER NUMERICAL RELATIONSHIPS.

Centigrade Temperatures to Fahrenheit. Multiply by 9/5 and add 32. [§2.]

Fahrenheit Temperatures to Centigrade. Subtract 32 and then multiply by 5/9. [§2.]

Centigrade Temperatures to Absolute. Add 273. [§28.]

Linear Expansion. $L_t = L_o (1 + \alpha t)$. [§10.]

Cubical Expansion. $V_t = V_o (1 + ct)$. [§10.]

Cubical and Linear Expansion. $C = 3\alpha$. [§9.]

Real and Apparent Expansion. $C_r = C_a + C_g$. [§18.]

Densities at Different Temperatures. $D_o = D_t (1 + ct)$.

Boyle's and Charles' Laws. $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$ [§30.]

Relative Humidity and Vapour Pressure.

$$\text{Relative humidity} = \frac{\text{max. pressure of vapour at dew point}}{\text{max. pressure at existing temp.}}$$

Mechanical Equivalent of Heat.

1 British Thermal Unit = 778 foot-pounds.

1 calorie = 4.26×10^4 gram-centimetres.

APPENDIX II. LIGHT.

(a) DEFINITIONS.

Accommodation (or **Accommodative Power**). The power possessed by the crystalline lens of the eye of altering its curvature, so that objects at different distances may give sharply defined images on the retina.

The change of curvature is brought about by the action of the *ciliary muscle.*

Achromatic Lens. A lens designed to correct chromatic aberration (q.v.)

Annular Eclipse. A peculiar kind of partial eclipse of the sun, in which the moon shuts off the light from every part of the sun except the edge of the disc ; thus only a circle of light is seen.

Candle-power. The candle-power of a source of light is the number of standard candles to which it is equivalent.

Candle (Standard). A sperm candle weighing one-sixth of a pound and burning away at the rate of 120 grains per hour.

Centre of Curvature. The centre of curvature of a concave or convex spherical mirror is the centre of the sphere of which the mirror forms a part (fig. 151).

Chromatic Aberration. A defect in a lens consisting in the production of a coloured fringe round the image. It is the result of *dispersion* (q.v.)

Complementary Colours. Two colours which, when combined, give the sensation of white.

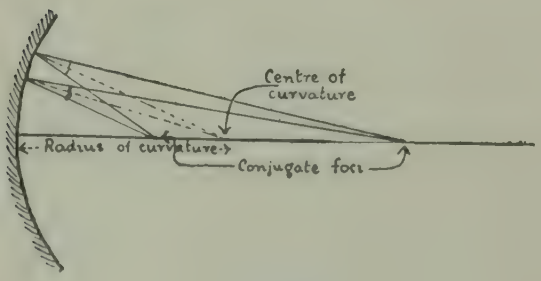

Fig. 151

Conjugate Foci. Two points in the neighbourhood of a mirror or lens, in such positions that if an object is placed at one point its image is formed at the other, and vice versa (fig. 151).

Critical Angle. Suppose a ray of light is travelling from one medium into an optically rarer one (i.e., so that the angle of refraction is greater than the angle of incidence). Then that angle of incidence, for which the corresponding angle of refraction is 90°, is known as the **critical angle.** If a ray is incident at an angle greater than the critical angle, it undergoes **total reflexion** at the surface of separation of the two media (fig. 152).

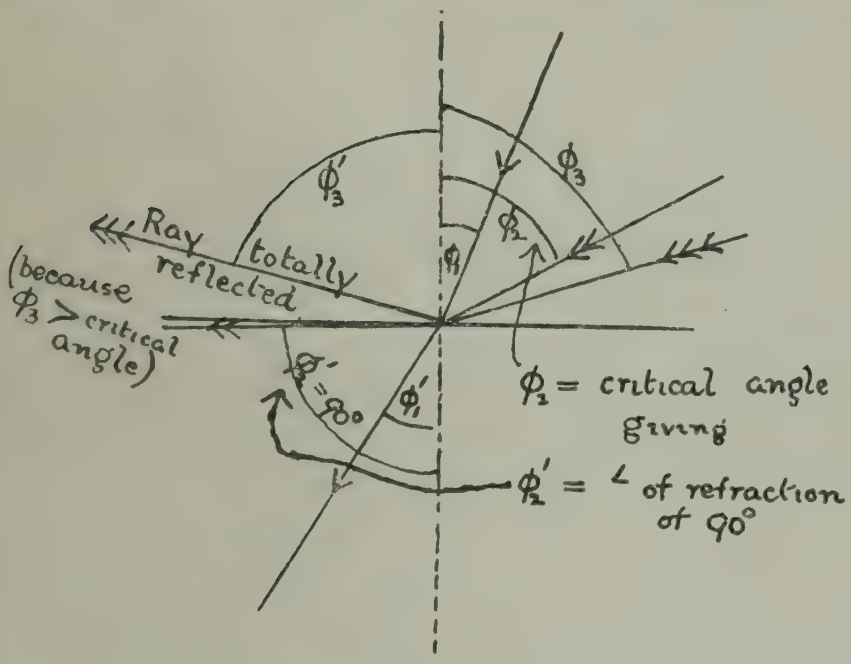

Fig. 152.

Deviation. The change in direction which a ray of light undergoes in passing from one medium to another. It is measured by finding the angle between the original and final directions of the ray (fig. 96).

Dispersion. The decomposition of white light, usually by a glass prism, into constituents of different colours (i.e., of different wave lengths). The decomposition takes place because waves of different length have different refractive indices (fig. 113).

Focal Length (*of a lens*). The distance from the optical centre of the lens to the focus (fig. 153).

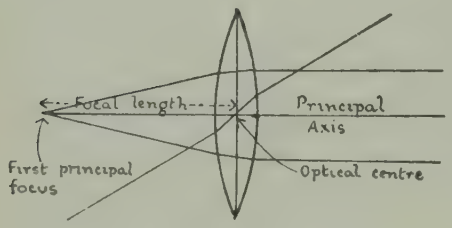

Fig. 153.

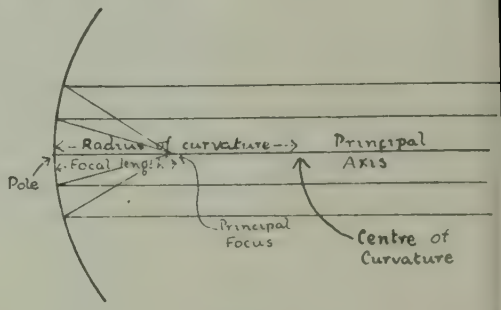

Fig. 154.

(*of a mirror*). The distance from the pole of the mirror to its principal focus (fig. 154).

Foci, Conjugate. (See **Conjugate Foci.**)

Focus (*of a lens*). Rays of light incident on a concave or convex lens in a direction parallel to the principal axis will after refraction (and being produced if necessary) pass through a point on that axis known as the **first principal focus** (fig. 153).

If the rays actually pass through the focus it is said to be **real,** but if it is necessary to produce them it is said to be **virtual** (figs. 75, 76).

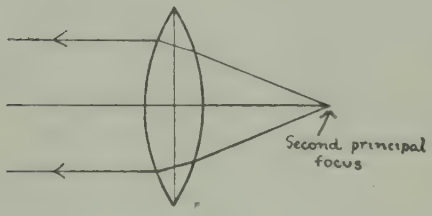

Fig. 155.

The **second principal focus** is a point on the principal axis of a convex lens such that rays of light diverging from it become parallel after passing through the lens (fig. 155).

(*of a mirror*). Rays of light incident on a spherical mirror in a direction parallel to the principal axis will after reflexion (and being produced if necessary) pass through a point on that axis known as the **focus** of the mirror (fig. 154).

If the rays actually pass through the focus it is said to be **real,** but if it is necessary to produce them it is said to be **virtual** (figs. 75, 76).

Hypermetropia. (See **Long Sight.**)

Illuminating Power. The illuminating power of a source of light is a quantity which relates to *the source itself* (contrast "intensity of illumination"). It is measured theoretically by the total light energy emitted by the source in 1 second, and practically by the number of standard candles which would be required to produce the same lighting effect as the source.

Intensity of Illumination. The intensity of illumination at a point due to a source of light, means the lighting effect of the source *at that point.* It is measured, theoretically, by the light energy which crosses unit area at right angles, in the neighbourhood of the point, in 1 second; and practically by comparison with the lighting effect of a standard candle at a distance of 1 foot.

Lateral Inversion. The effect produced by reflexion in a plane mirror, points on the left of the object being on the right of the image and vice versa (fig. 67).

Lens. A portion of a refracting medium bounded by surfaces of a definite geometrical shape (fig. 100).

Long Sight. A defect of vision which is marked by an inability to see small objects distinctly when they are placed as near to the eye as 10 or 12 inches (the "nearest distance of distinct vision" for a person of normal sight) (fig. 125).

Magnification. In general, magnification is the ratio of the length of the image to that of the object.

When the object is distant (e.g., in the case of a telescope) the apparent length is measured by the *visual angle* (i.e., the angle which the length of the object subtends at the eye). The image can be altered in position until it is in the most favourable position (i.e., at the nearest distance of distinct vision). The magnification is then the ratio x/y, when x is the angle subtended by the image at the nearest distance of distinct vision, and y is the angle subtended by the object in its actual position.

Myopia. (See **Short Sight.**)

Normal. A straight line drawn at right angles to the surface of separation of two media (fig. 156).

Optical Centre. Suppose a ray of light passes through a lens in such a way that its final direction is parallel to its original one. Let A be the point at which it enters the lens, B that at which it leaves it. Then the point in which AB intersects the principal axis of the lens is known as the optical centre (fig. 153).

Penumbra. A region of partial shadow (fig. 54).

Photometry. The measurement, or comparison, of the illuminating powers of various sources of light (cp. fig. 57).

Pole. The pole of a curved mirror is the middle point of the reflecting surface (fig. 154).

Principal Axis (*of a spherical mirror*). The straight line joining the centre of curvature to the pole (fig. 154)

(*of a lens.*) The straight line joining the centres of curvature of the two bounding spherical surfaces (fig. 101).

Principal Focus. (See **Focus.**)

Radius of Curvature. The radius of curvature of a concave or convex spherical mirror is the radius of the sphere of which the mirror forms a part, *or* it is the distance from the centre of curvature to any point on the mirror (fig. 154).

Real Image. One formed by the actual intersection of reflected or refracted rays of light, and which can therefore be received upon a screen (fig. 104).

If the rays intersect only on being produced, they will give rise to a **virtual image,** which cannot be received upon a screen (fig. 105).

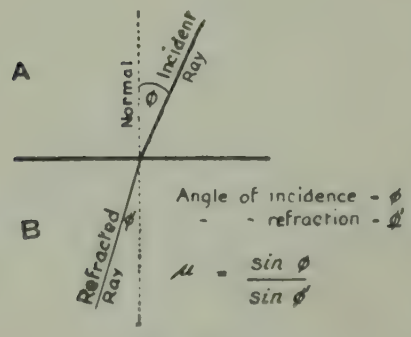

Fig. 156.

Refractive Index. Suppose a ray of light is passing from a medium, A, to another one, B, the angle of incidence at the surface of separation being ϕ and the angle of refraction ϕ'. Then the constant ratio $\sin \phi / \sin \phi'$ is known as the *index of refraction* (or *refractive index*) from A to B, and is written as ${}_A\mu_B$ (fig. 156).

If the first medium is air (or, strictly speaking, vacuum) the constant ratio is spoken of simply as the *refractive index* of the second medium, and is written as μ.

Short Sight. A defect of vision marked by an inability to see objects distinctly except when they are placed very close to the eye (fig. 123).

Spectrum (visible). The coloured band which is produced when a beam of light is received upon a screen after undergoing dispersion (q.v.) (fig. 113).

Standard Candle. (See **Candle.**)

Total Reflexion. (See under **Critical Angle.**)

Umbra. A region of complete shadow (figs. 54, 55).

Virtual Image. (See under **Real Image.**)

Visual Angle. The angle which an object subtends at the eye. It is this angle which determines the apparent size of the object (fig. 157).

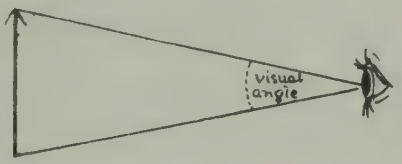

Fig. 157.

(b) LAWS.

Law of Inverse Squares. The intensity of illumination at any point is inversely proportional to the square of the distance of that point from the source of light.

Laws of Reflexion.

(1.) The reflected ray lies in the plane containing the incident ray and the normal.

(2.) The angle of reflexion is equal to the angle of incidence.

Laws of Refraction.

(1.) The refracted ray lies in the plane containing the incident ray and the normal.

(2.) For a given pair of media, and for light of a given wave length, a constant number is obtained on dividing the sine of the angle of incidence by the sine of the angle of refraction.

(c) FORMULÆ AND OTHER NUMERICAL RELATIONSHIPS.

Rotation of mirror (α) *and of reflected ray* (θ). $\theta = 2\alpha$.

Focal length of mirror and radius of curvature. $r = 2f$.

Positions of image and object (*mirror*). $1/v + 1/u = 1/f$.

Magnification (*mirror*).

$$\text{Magnification} = \frac{\text{length of image}}{\text{length of object}} = -v/u.$$

If magnification ($-v/u$) is positive } image is erect and
i.e., if v/u is negative } vice versa [§131.]

Index of refraction (*definition*). $\mu = \sin\phi/\sin\phi'$.

Transparent medium. $\mu = \dfrac{\text{real depth}}{\text{apparent depth}}$

Critical angle (ϕ). $\text{Sin}\ \phi = 1/\mu$.

Minimum deviation by prism. $\mu = \dfrac{\sin\frac{1}{2}(D+A)}{\sin\frac{1}{2}A}$ [§143.]

Positions of image and object (*lens*). $1/v - 1/u = 1/f$. [§150.]

Magnification (*lens*).

$$\text{Magnification} = \frac{\text{length of image}}{\text{length of object}} = v/u.$$

If v/u is positive (i.e., if object and image are on same side of lens), image is *erect* and vice versa.

Magnification—various optical instruments.

(i.) *Simple and compound microscope.*

$$\text{Magnification} = \frac{\text{length of image}}{\text{length of object}}$$

In case of simple microscope, this is approximately equal to $\dfrac{\text{nearest distance of distant vision}}{\text{focal length of lens}}$

(ii.) *Astronomical and Galilean telescope.*

$$\text{Magnification} = \frac{\text{focal length of object glass}}{\text{focal length of eye-piece}}$$

APPENDIX III. SOUND.

(a) DEFINITIONS.

Amplitude. (See under **Vibration.**)

Antinodes. Those points, in a medium subject to stationary undulation, at which the displacement is always a maximum.

Beats. A peculiar throbbing effect, the throbs or *beats* succeeding each other at very short but equal intervals of time. The effect is produced by two sources of sound, of nearly equal frequency, vibrating simultaneously, and it can be shown that the frequency of the beats is equal to the difference between the frequencies of the two sources.

Displacement Curve. A curve which shows the displacement, at a given instant, of the various points in a medium through which longitudinal or transverse waves are passing.

It can be shown that the same curve also represents the displacement, at successive instants, of any one point in such a medium.

Doppler's Principle. If a source of sound is in motion, the waves in front of it are shorter, and those behind it longer, than they would have been if the source had been stationary.

Frequency of Vibration. (See under **Vibration.**)

Fundamental. The lowest note which can be produced when a portion of a medium is subject to stationary undulation. The higher notes which may be produced are known as **harmonics** or **overtones.**

Harmonics. (See above under **Fundamental.**)

Interference. The principle that if two (or more) systems of waves are travelling through a given medium, each produces the same effect on the medium as if no other system were present.

Nodes. Those points, in a medium subject to stationary undulation, at which the displacement is always zero (fig. 149).

Overtones. (See under **Fundamental.**)

Period of Vibration. (See under **Vibration.**)

Phase. The phase of a vibrating particle is the fraction t'/t, where t' is the time that has elapsed since the particle last passed through its mean position in the positive direction, and t is its period of vibration.

Pitch. The pitch of a note is its property of imparting to the ear a sensation of highness or lowness, and can be shown to depend on the frequency of vibration of the body producing the note.

Quality. The property by which we can distinguish two sounds that are of the same pitch and the same loudness. It depends on the number and intensity of the overtones present or (as these determine the shape of the displacement curve) we may say that quality depends on the shape of the displacement curve.

Resonance. The production of strong vibration in a body by so timing a succession of impulses that the interval between them coincides with one of the natural periods of vibration of the body.

Vibration. The motion of a particle which repeatedly makes the same (or nearly the same) movement and takes the same (or nearly the same) time in doing so.

The **period of vibration** is the time required for one complete movement, and the **frequency of vibration** is the number of complete movements made in 1 second.

The **amplitude of vibration** is the distance from the mean position of the vibrating particle to the furthest point reached on one side or the other.

Wave Length. The distance between two consecutive particles which are in the same phase of vibration.

In the case of stationary undulation, wave length may be defined as the distance between two consecutive nodes or between two consecutive antinodes.

(b) LAWS.

Laws of Stretched Strings. When a string is vibrating in its simplest mode, the frequency of the note emitted varies :—

(i.) Inversely as the length ;
(ii.) Directly as the square root of the stretching force ;
(iii.) Inversely as the square root of the mass per unit length.

(c) FORMULÆ AND OTHER NUMERICAL RELATIONSHIPS.

Wave length, frequency and velocity. $v = n\lambda$. [§178.]

Velocity of sound. $v = \sqrt{\gamma \frac{P}{D}}$ where for most gases $\gamma = 1.41$. [§188.]

Effect of pressure and temperature. Velocity of sound is
(i.) Independent of pressure ;
(ii.) Proportional to square root of absolute temperature. [§189.]

Beats. Frequency of beats = difference between the frequencies of the two sounding bodies. [§192.]

Simplest conditions for resonance.
(i.) *Closed tube.* Sound travels four times length of tube while fork makes one complete vibration. [§195.]
(ii.) *Open tube.* Sound travels twice length of tube while fork makes one complete vibration. [§196.]

Velocity of sound by resonance (closed tube). $v = n\lambda$.
(n = frequency of fork, λ = wave length = 4 times length of tube.)

Velocity of transverse wave along wire. $v = \sqrt{\frac{T}{m}}$ [§200.]

Stretched string vibrating in simplest mode. $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$ [§207.]

Relative frequencies of notes of major diatonic scale (doh, ray, mi, etc.) 24 : 27 : 30 : 32 : 36 : 40 : 45 : 48.

RECIPROCALS AND SINES

TABLES OF RECIPROCALS OF NUMBERS FROM 1 TO 99 AND SINES OF ANGLES FROM 1° TO 90°.

Number or Degrees	Reciprocal	Sine	Number or Degrees	Reciprocal	Sine	Number or Degrees	Reciprocal	Sine
1	1	.0175	34	.0294	.5592	67	.0149	.9205
2	.5000	.0349	35	.0286	.5736	68	.0147	.9272
3	.3333	.0523	36	.0278	.5878	69	.0145	.9336
4	.2500	.0698	37	.0270	.6018	**70**	.0143	.9397
5	.2000	.0872	38	.0263	.6157	71	.0141	.9455
6	.1667	.1045	39	.0256	.6293	72	.0139	.9511
7	.1429	.1219	**40**	.0250	.6428	73	.0137	.9563
8	.1250	.1392	41	.0244	.6561	74	.0135	.9613
9	.1111	.1564	42	.0238	.6691	75	.0133	.9659
10	.1000	.1737	43	.0233	.6820	76	.0132	.9703
11	.0909	.1908	44	.0227	.6947	77	.0130	.9744
12	.0833	.2079	45	.0222	.7071	78	.0128	.9782
13	.0769	.2250	46	.0217	.7193	79	.0127	.9816
14	.0714	.2419	47	.0213	.7314	**80**	.0125	.9848
15	.0667	.2588	48	.0208	.7431	81	.0123	.9877
16	.0625	.2756	49	.0204	.7547	82	.0122	.9903
17	.0588	.2924	**50**	.0200	.7660	83	.0120	.9926
18	.0556	.3090	51	.0196	.7772	84	.0119	.9945
19	.0526	.3256	52	.0192	.7880	85	.0118	.9962
20	.0500	.3420	53	.0189	.7986	86	.0116	.9976
21	.0476	.3584	54	.0185	.8090	87	.0115	.9986
22	.0455	.3746	55	.0182	.8192	88	.0114	.9994
23	.0435	.3907	56	.0179	.8290	89	.0112	.9999
24	.0417	.4067	57	.0175	.8387	**90**	.0111	1.0000
25	.0400	.4226	58	.0172	.8481	91	.0110	
26	.0385	.4384	59	.0169	.8572	92	.0109	
27	.0370	.4540	**60**	.0167	.8660	93	.0108	
28	.0357	.4695	61	.0164	.8746	94	.0106	
29	.0345	.4848	62	.0161	.8830	95	.0105	
30	.0333	.5000	63	.0159	.8910	96	.0104	
31	.0323	.5150	64	.0156	.8988	97	.0103	
32	.0313	.5299	65	.0154	.9063	98	.0102	
33	.0303	.5446	66	.0152	.9136	99	.0101	

[The above table is reprinted from Section V. of Mr. S. E. Brown's *Experimental Science*, by kind permission of the Cambridge University Press.]

Answers to Numerical Examples.

CHAPTER I.

(1.) 674.6 ; —38.2 ; 59 ; —459.4 ; 172 4.
(2) 15.6 ; —40 ; 114.4 ; —17.8 ; 36.9.
(3.) 15.
(4.) 36.
(5.) (i.) .000012 ; (ii.) .0000067.
(6.) .102 yds. or 3.67 ins.
(7.) 1005.36 c.c.
(8.) 7.80.
(9.) 40° C. ; —5° C ; 50° F. ; 5° F.
(10.) 211.3° F.
(11.) 104° F.
(13.) 29.728 ins.
(14.) Between .000011 and .000015.
(16.) 40° C. ; 76.9° C.
(17.) 126.7 cms.

CHAPTER II.

(1.) .000023.
(2.) .000489.
(3.) .000185.
(4.) 293° A. ; 373° A. ; 233° A.
(5.) 309.7° A. ; 289.7° A. ; 233° A.
(6.) (i.) 13.66 l. ; (ii.) 7.471.
(7.) 531.6 c.c.
(8.) 23.94 l.
(9.) .00367.
(10.) .00373.
(11.) 150.0° C.
(12.) 985.5 gms.
(14.) .03 c.c. ; 1.875 c.c.
(19.) 689 mm.
(20.) 783 mm.

CHAPTER III.

(12.) 770 mm.

CHAPTER IV.

(1.) 100 gms.
(3.) .51.
(4.) 6.6 mins.
(5.) W.E. = 5.6 gms. ; Sp. heat = .126.
(6.) (i.) 1.17 gms. ; (ii.) .1.
(7.) 645° C.
(9.) .117.
(10.) 35° C.
(12.) 1.818 : 1.
(13.) 82.3.
(14.) 13.28 mm.
(16.) 64.8 gms.
(17.) 28.1° C.
(18.) 540.
(19.) 195,700 cals.
(21.) .67.
(23.) 11.6° F.
(27.) 7.07 %.
(28.) 1166.7.
(29.) 4.27° F.

CHAPTER VI.

(7.) 125 : 64.
(9.) $\sqrt{2}$ ft. (= 1.41 ft. or 1 ft. 5 ins.)
(10.) 62.5 cms.
(13.) 126.5 cms.
(14.) 1.216 : 1.
(21.) 2.86° or 2° 52′.
(22.) 55°.

CHAPTER VII.

(3.) (a) 30 cms. ; (b) 10 cms. from mirror.
(4.) 1.10 ft. from candle (11.10 ft. from wall).
(5.) $6\frac{2}{3}$ ins. ; $3\frac{1}{3}$ ins. from mirror.
(6.) $3\frac{1}{3}$ cms. ; 5 cms.

(7.) 4 ft. from mirror.
(8.) 1 ft. from candle ; $\frac{10}{11}$ ft. (= 10.9 ins.)
(9) 2 ft. (real image) ; 1 ft. (virtual).
(13) $13\frac{1}{3}$ cms.
(14.) $3\frac{1}{3}$ ins. or 10 ins.
(15.) (a) 15 cms. ; (b) 5 cms. from mirror.
(16.) 30 cms. ; 15 cms.

CHAPTER VIII.

(2.) $2\frac{2}{3}$ cms.
(10.) 18° 35′.
(13.) $52\frac{1}{2}$° ; $37\frac{1}{2}$°.
(14.) 47°.
(17.) 37.2°.
(18.) 38.9° ; 37.2°.

CHAPTER IX.

(3.) Convex ; $6\frac{3}{4}$ ins. ; 2 ft. 3 ins. from screen.
(4.) (a) Real image, 30 cms. from lens.
(b) Image at ∞.
(c) Virtual image at focus.
(5.) (a) $4\frac{4}{5}$ ft. from screen ; $f = -\frac{24}{25}$ ft.
(b) 8 ft. from screen ; $f = 1\frac{3}{5}$ ft.
(6.) 1 ft.
(7.) (a) $13\frac{1}{3}$ cms. from lens ;
(b) $6\frac{2}{3}$ cms. from lens.
(8.) 2 ins. in front of lens of shorter focal length.
(9.) Convex ; $18\frac{3}{4}$ cms. ; 25 cms. from object, 75 cms. from screen.
(11.) Convex, 20 cms. ; concave, 60 cms.
(12.) 33° 44′.
(13.) 2 ft. from wall ; $1\frac{1}{3}$ ft.

CHAPTER X.

(10.) 2 ins. from object.
(14.) Convex lens ; $f = 13\frac{11}{13}$ ins.
(15.) Convex ; f = 2 ft.
(16.) Convex ; $f = 27\frac{3}{11}$ cms.
(17.) 4 cms. ; m = 5.
(18.) Concave ; f = 100 cms. ; 7.53 cms.
(22.) 7 cms. from convex lens.
(23.) 5.

CHAPTER XI.

(5.) 9.
(15.) 280 ft.
(16.) 341.3 m. per sec.
(17.) 252 and 260.
(21.) 273.
(23.) 33,472 cms./sec.

CHAPTER XII.

(7.) 9 : 16 increase.
(8.) 4 : 9 increase.
(9.) $\sqrt{2}$: 1 decrease.
(10.) 5 lbs.
(14.) 33.3 cms. (end portions 40 cms. and 26.7 cms. respectively).

No	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						4	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	36
11	0414	0453	0492	0531	0569						4	8	12	15	19	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	19	22	26	30	33
12	0792	0828	0864	0899	0934						3	7	11	14	18	21	25	28	32
						0969	1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	7	10	13	16	20	23	26	30
						1303	1335	1367	1399	1430	3	6	10	13	16	19	22	25	29
14	1461	1492	1523	1553	1584						3	6	9	12	15	19	22	25	28
						1614	1644	1673	1703	1732	3	6	9	12	15	17	20	23	26
15	1761	1790	1818	1847	1875						3	6	9	11	14	17	20	23	26
						1903	1931	1959	1987	2014	3	6	8	11	14	17	19	22	25
16	2041	2068	2095	2122	2148						3	5	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	16	18	21	23
17	2304	2330	2355	2380	2405						3	5	8	10	13	15	18	20	23
						2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	16	19	21
						2672	2695	2718	2742	2765	2	5	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8

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No	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

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N.B.— (i.) *The numbers refer to* **paragraphs,** *not to* **pages.**
(ii.) *Many definitions are given in the Appendixes, pp.* 208-222

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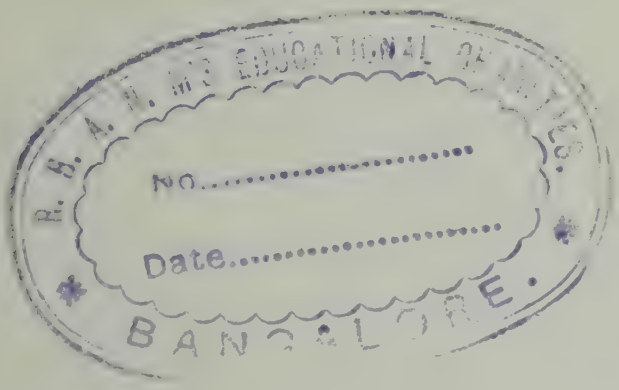

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